

# Theory of Computer Science

## C8. Type-1 and Type-0 Languages: Closure & Decidability

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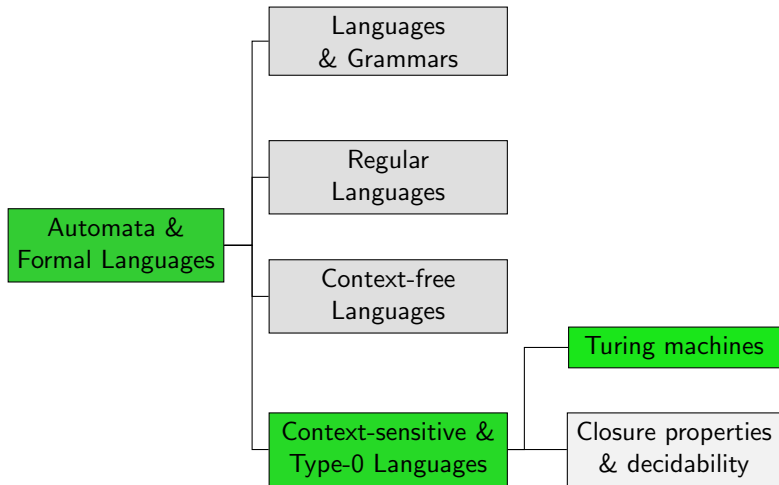
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## C8.1 Turing Machines vs. Grammars

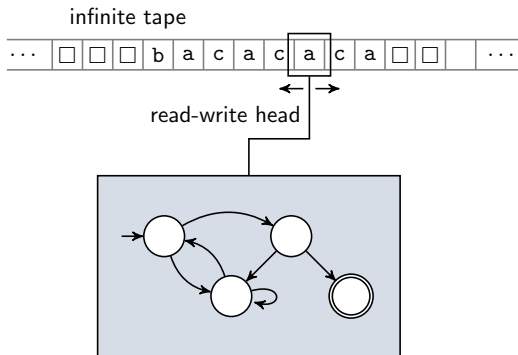
## C8.2 Closure Properties and Decidability

# Overview



# C8.1 Turing Machines vs. Grammars

# Reminder: Turing Machines – Conceptually



# Reminder: Nondeterministic Turing Machine

## Definition (Nondeterministic Turing Machine)

A nondeterministic **Turing machine (NTM)** is given by a 7-tuple  $M = \langle Q, \Sigma, \Gamma, \delta, q_0, \square, E \rangle$  with:

- ▶  $Q$  finite non-empty set of **states**
- ▶  $\Sigma \neq \emptyset$  finite **input alphabet**
- ▶  $\Gamma \supset \Sigma$  finite **tape alphabet**
- ▶  $\delta : (Q \setminus E) \times \Gamma \rightarrow \mathcal{P}(Q \times \Gamma \times \{L, R, N\})$  **transition function**
- ▶  $q_0 \in Q$  **start state**
- ▶  $\square \in \Gamma \setminus \Sigma$  **blank symbol**
- ▶  $E \subseteq Q$  **end states**

# One Automata Model for Two Grammar Types?

Don't we need  
different automata models for  
context-sensitive and type-0  
languages?



Picture courtesy of stockimages / FreeDigitalPhotos.net

# Linear Bounded Automata: Idea

- ▶ **Linear bounded automata** are NTMs that may only use the **part of the tape occupied by the input word**.
- ▶ one way of formalizing this: NTMs where blank symbol may never be replaced by a different symbol



# Linear Bounded Turing Machines: Definition

## Definition (Linear Bounded Automata)

An NTM  $M = \langle Q, \Sigma, \Gamma, \delta, q_0, \square, E \rangle$

is called a **linear bounded automaton (LBA)**

if for all  $q \in Q \setminus E$  and all transition rules  $\langle q', c, y \rangle \in \delta(q, \square)$   
we have  $c = \square$ .

**German:** linear beschränkte Turingmaschine

# LBAs Accept Type-1 Languages

## Theorem

*The languages that can be accepted by linear bounded automata are exactly the context-sensitive (type-1) languages.*

Without proof.

proof sketch for grammar  $\Rightarrow$  NTM direction:

- ▶ computation of the NTM follows the production of the word in the grammar **in opposite order**
- ▶ accept when only start symbol (and blanks) are left on the tape
- ▶ because language is context-sensitive, we never need additional space on the tape (empty word needs special treatment)

# NTMs Accept Type-0 Languages

## Theorem

*The languages that can be accepted by nondeterministic Turing machines are exactly the type-0 languages.*

Without proof.

proof sketch for grammar  $\Rightarrow$  NTM direction:

- ▶ analogous to previous proof
- ▶ for grammar rules  $w_1 \rightarrow w_2$  with  $|w_1| > |w_2|$ , we must “insert” symbols into the existing tape content; this is a bit tedious, but not very difficult

# Deterministic Turing Machines

## Definition (Deterministic Turing Machine)

A **deterministic Turing machine (DTM)** is a Turing machine

$M = \langle Q, \Sigma, \Gamma, \delta, q_0, \square, E \rangle$  with

$\delta : (Q \setminus E) \times \Gamma \rightarrow Q \times \Gamma \times \{L, R, N\}$ .

**German:** deterministische Turingmaschine

# Deterministic Turing Machines vs. Type-0 Languages

## Theorem

*For every type-0 language  $L$  there is a deterministic Turing machine  $M$  with  $\mathcal{L}(M) = L$ .*

Without proof.

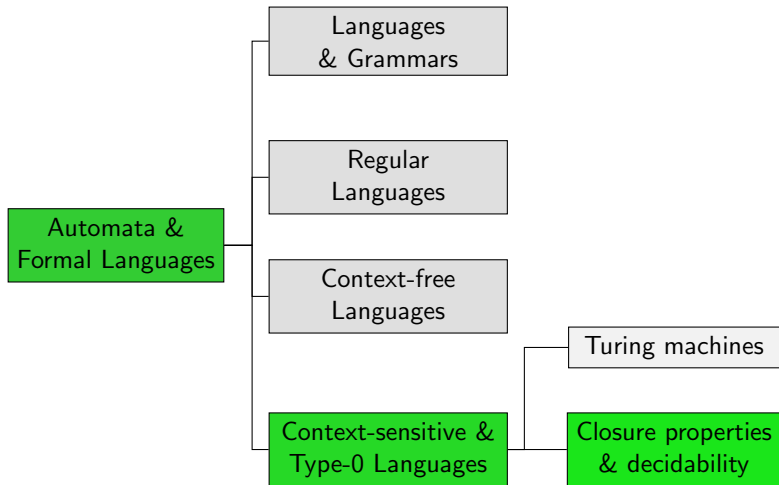
proof sketch:

- ▶ Let  $M'$  be an NTM with  $\mathcal{L}(M') = L$ .
- ▶ It is possible to construct a DTM that systematically searches for an accepting configuration in the computation tree of  $M'$ .

**Note:** It is an open problem whether an analogous theorem holds for type-1 languages and deterministic LBAs.

## C8.2 Closure Properties and Decidability

# Overview



# Closure Properties

	Intersection	Union	Complement	Concatenation	Star
Type 3	Yes	Yes	Yes	Yes	Yes
Type 2	No	Yes	No	Yes	Yes
Type 1	Yes <sup>(2)</sup>	Yes <sup>(1)</sup>	Yes <sup>(2)</sup>	Yes <sup>(1)</sup>	Yes <sup>(1)</sup>
Type 0	Yes <sup>(2)</sup>	Yes <sup>(1)</sup>	No <sup>(3)</sup>	Yes <sup>(1)</sup>	Yes <sup>(1)</sup>

## Proofs?

- (1) proof via grammars, similar to context-free cases
- (2) without proof
- (3) proof in later chapters (part D)



# Decidability

	Word problem	Emptiness problem	Equivalence problem	Intersection problem
Type 3	Yes	Yes	Yes	Yes
Type 2	Yes	Yes	No	No
Type 1	Yes <sup>(1)</sup>	No <sup>(3)</sup>	No <sup>(2)</sup>	No <sup>(2)</sup>
Type 0	No <sup>(4)</sup>	No <sup>(4)</sup>	No <sup>(4)</sup>	No <sup>(4)</sup>

Proofs?

- (1) same argument we used for context-free languages
- (2) because already undecidable for context-free languages
- (3) without proof
- (4) proofs in later chapters (part D)

# Summary

- ▶ **Turing machines** accept exactly the **type-0** languages. This is also true for **deterministic Turing machines**.
- ▶ **Linear bounded automata** accept exactly the **context-sensitive** languages.
- ▶ The context-sensitive and type-0 languages are **closed** under **almost all** usual operations.
  - ▶ exception: **type-0 not closed** under **complement**
- ▶ For context-sensitive and type-0 languages **almost no problem is decidable**.
  - ▶ exception: **word problem** for **context-sensitive** lang. decidable

# What's Next?

contents of this course:

A. **background** ✓

▷ mathematical foundations and proof techniques

B. **logic** ✓

▷ How can knowledge be represented?  
How can reasoning be automated?

C. **automata theory and formal languages** ✓

▷ What is a computation?

D. **Turing computability**

▷ What can be computed at all?

E. **complexity theory**

▷ What can be computed efficiently?

F. **more computability theory**

▷ Other models of computability