Theory of Computer Science

G. Röger
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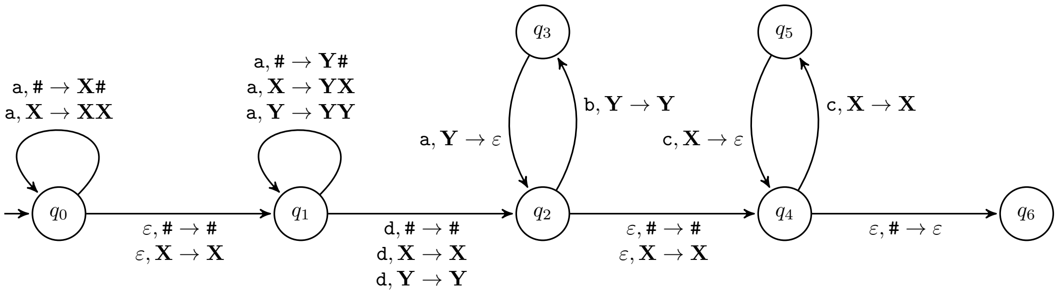
University of Basel
Computer Science

Exercise Sheet 7 — Solutions

Exercise 7.1 (Push-down Automata; 2+2 marks)

Consider the push-down automaton (PDA) $M = \langle Q, \Sigma, \Gamma, \delta, q_0, \texttt{\#}\rangle$ with $Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}$, $\Sigma = \{\texttt{a}, \texttt{b}, \texttt{c}, \texttt{d}\}$, $\Gamma = \{\mathbf{X}, \mathbf{Y}, \texttt{\#}\}$, and the following transition function δ:



(a) Show that the automaton accepts the word *aaadbabacc* by specifying an accepting sequence of configurations.

Solution:

$$
\begin{aligned}
\langle q_0, \texttt{aaadbabacc}, \texttt{\#}\rangle &\vdash_M \langle q_0, \texttt{aadbabacc}, \mathbf{X}\texttt{\#}\rangle \\
&\vdash_M \langle q_1, \texttt{aadbabacc}, \mathbf{X}\texttt{\#}\rangle \\
&\vdash_M \langle q_1, \texttt{adbabacc}, \mathbf{YX}\texttt{\#}\rangle \\
&\vdash_M \langle q_1, \texttt{dbabacc}, \mathbf{YYX}\texttt{\#}\rangle \\
&\vdash_M \langle q_2, \texttt{babacc}, \mathbf{YYX}\texttt{\#}\rangle \\
&\vdash_M \langle q_3, \texttt{abacc}, \mathbf{YYX}\texttt{\#}\rangle \\
&\vdash_M \langle q_2, \texttt{bacc}, \mathbf{YX}\texttt{\#}\rangle \\
&\vdash_M \langle q_3, \texttt{acc}, \mathbf{YX}\texttt{\#}\rangle \\
&\vdash_M \langle q_2, \texttt{cc}, \mathbf{X}\texttt{\#}\rangle \\
&\vdash_M \langle q_4, \texttt{cc}, \mathbf{X}\texttt{\#}\rangle \\
&\vdash_M \langle q_5, \texttt{c}, \mathbf{X}\texttt{\#}\rangle \\
&\vdash_M \langle q_4, \varepsilon, \texttt{\#}\rangle \\
&\vdash_M \langle q_6, \varepsilon, \varepsilon\rangle
\end{aligned}
$$

(b) What language does this automaton accept?

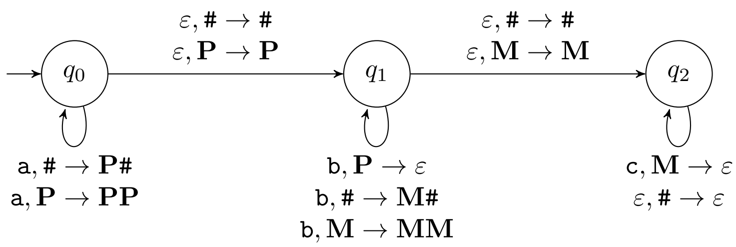
Solution:

$\mathcal{L}(M) = \{\texttt{a}^i\texttt{a}^j\texttt{d}(\texttt{ba})^j\texttt{c}^{2i} \in \Sigma^* \mid i, j \geq 0\}$

Exercise 7.2 (Push-down Automata; 2 marks)

Consider language $L = \{\texttt{a}^i\texttt{b}^j\texttt{c}^k \mid i, j, k \geq 0, j = i + k\}$. Specify a push-down automaton (PDA) that accepts L. Remember to specify a complete description with all components. You can specify the transition function δ as a transition diagram.

Solution:
$M = \langle\{q_0, q_1, q_2\}, \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}, \{\mathbf{M}, \mathbf{P}, \#\}, \delta, q_0, \#\rangle$, where δ is defined as follows:



Note: This exercise was part of the exam in 2018 and gave 5 out of 80 marks there.

Exercise 7.3 (Turing machines; 2 marks)
A *deterministic* Turing machine $M = \langle Q, \Sigma, \Gamma, \delta, q_0, \Box, E\rangle$ is defined like a non-deterministic Turing machine, but with $\delta : (Q \setminus E) \times \Gamma \to Q \times \Gamma \times \{\mathrm{L}, \mathrm{R}, \mathrm{N}\}$. This means that for every non-end state and every symbol from Γ there is exactly one transition (specifying the change of the state and on the tape and the movement of the head).

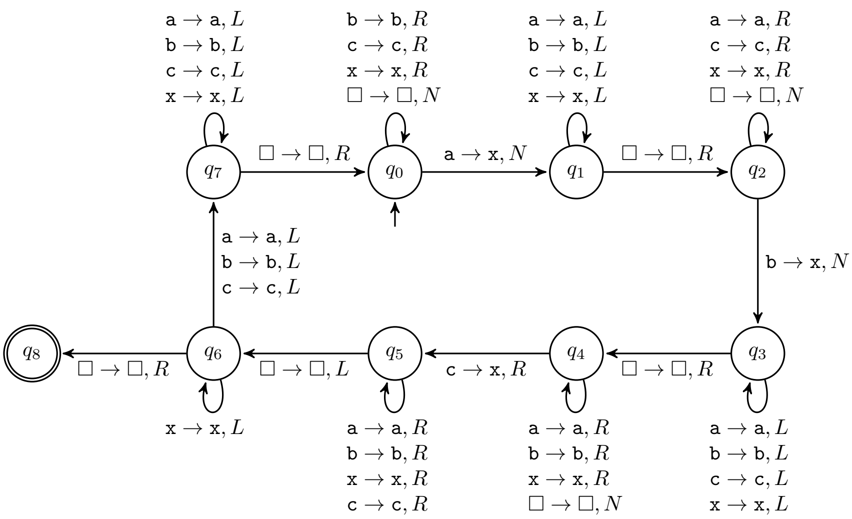
A Turing machine M that accepts the language

$$L = \{w \in \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}^+ \mid w \text{ contains the same number of } \mathtt{a}, \mathtt{b}, \text{ and } \mathtt{c}\}$$

works with the following loop:

M looks for an a on the tape and replaces it with an x. Then it searches a b on the tape and replaces it with an x. Then is searches a c on the tape and replaces it with an x. If any symbol is not found in this way, M enters an endless loop. Otherwise, it checks if all symbols on the tape have been replaced with x. If this is the case, it moves to an accepting state, otherwise the loop starts again.

Specify a transition diagram for a *deterministic* Turing machine with this behavior. Explain your solution by describing which part of the Turing machine is responsible for which part of the loop.
Solution:



State q_0 starts at the beginning of the input and reads the word from left to right until the first a. All symbols remain unchanged until the first a is replaced with an x at which time the TM

changes to state q_2. If state q_0 reaches the end of the word without reading an `a`, the TM stay in this state forever.
State q_1 moves the read head back to the beginning of the word and the TM transitions to state q_2. this state works analogously to q_0 and replaces the first `b` with an `x`. After that q_3 moves the head back to the beginning of the word (analogously to q_1) and q_4 replaces the first `c` with an `x` (analogously to q_0).
Now q_5 moves the head to the end of the input (analogously to q_1 but in the other direction). After that, state q_6 reads the word from right to left. If it only reads `x`, the TM transitions to the accepting state q_8. Otherwise, the TM transitions to state q_7, which moves the head back to the start of the input (analogously to q_1).