Theory of Computer Science

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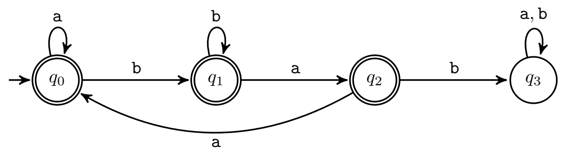
Computer Science

Exercise Sheet 5 — Solutions

Exercise 5.1 (DFA and NFA, 1.5+1.5 marks)

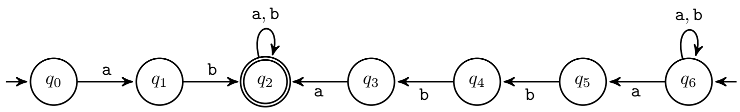
(a) Specify a deterministic finite automaton that accepts the language of all words over $\Sigma = \{\mathtt{a}, \mathtt{b}\}$ that do *not* contain `bab` (e.g., the word `ababa` is not contained).

Solution:



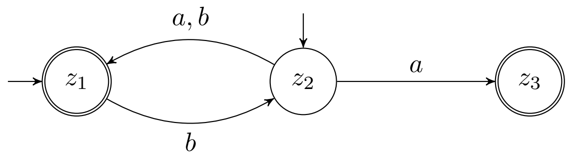
(b) Specify a non-deterministic finite automaton that accepts the language of those words over $\Sigma = \{\mathtt{a}, \mathtt{b}\}$ that start with `ab` or contain `abba`.

Solution:



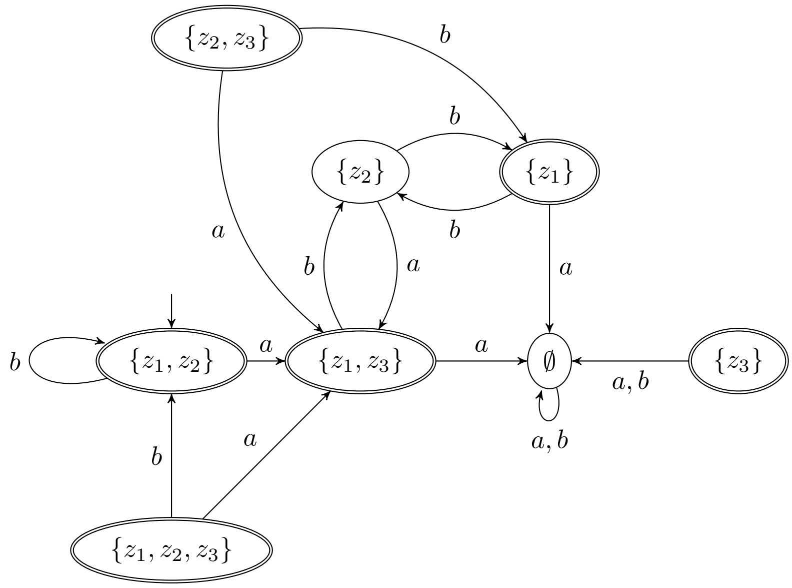
Exercise 5.2 (DFA and NFA, 2 marks)

Specify a DFA that is equivalent to the following NFA.

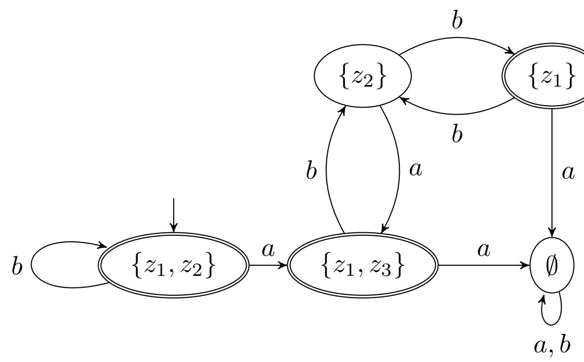


Solution:

The following DFA is constructed by following the algorithm from the lecture (slide 28/35 from handout version of slide set C2).



This DFA still contains some unreachable states. The solution is easier to follow if we remove them.



Exercise 5.3 (Regular Expressions, 2 marks)

Consider the following regular expressions over the alphabet $\Sigma = \{0, 1\}$. For each regular expression, specify two words that are in the corresponding language and two words that are not in the corresponding language.

(a) $110|1001$

(b) $1^*(01^*01^*)^*$

(c) $(0\varepsilon|1(0|1))(0|1)^*$

(d) $1(\varepsilon|0)|0\emptyset 1$

Solution:

(a) $L(110|1001) = \{110, 1001\}$
The words 110 and 1001 are in the language. Examples for words that are not in the language are 1 and 0110.

(b) $L(1^*(01^*01^*)^*) = \{w \mid\ w \text{ contains an even number of 0s}\}$
Examples for words in the language are 00 and 0101010. Examples for words that are not in the language are 10 and 010101.

(c) $L((0\varepsilon|1(0|1))(0|1)^*) = \Sigma^* \setminus \{\varepsilon, 1\}$
The words 0011 and 0001 are examples for words in the language. The words that are not in the language are ε and 1.

(d) $L(1(\varepsilon|0)|0\emptyset 1) = \{1, 10\}$
The words 1 and 10 are in the language. Examples for words that are not in the language are ε and 0.

Exercise 5.4 (Regular Expressions, 1 mark)

Specify a regular expression that describes the language

$$L = \{w \in \{0,1\}^* \mid |w| \geq 2, w \text{ ends with 0 and contains at most two 0s}\}.$$

Solution:

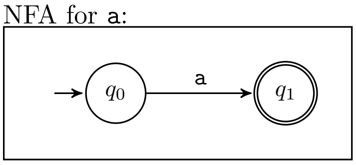
$L = \mathcal{L}(1^*(1|0)1^*0)$

Exercise 5.5 (NFAs for Regular Expressions; 2 Points)

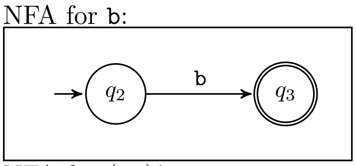
Construct an NFA for the regular expression $((\texttt{ab})^*|\texttt{a}^*)$ over the alphabet $\Sigma = \{\texttt{a}, \texttt{b}\}$. Use the construction rules from the lecture (chapter C3 slides 13–16 on handout version) and please specify all intermediate steps, i.e., NFAs for a, b, ab, $(\texttt{ab})^*$, $\texttt{a}^*$ and $((\texttt{ab})^*|\texttt{a}^*)$.

Solution:

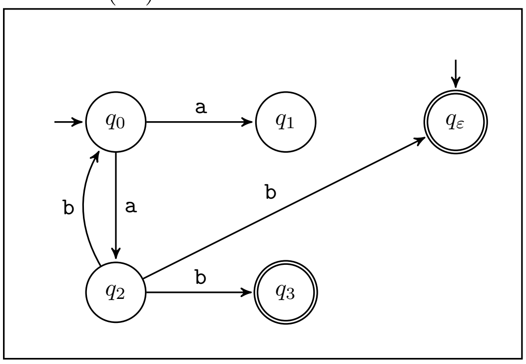
NFA for a:



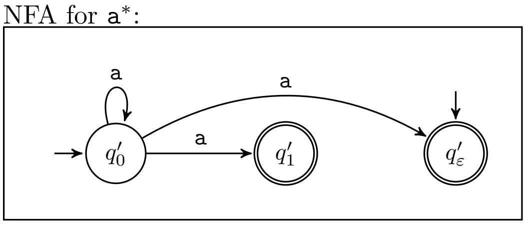
NFA for b:



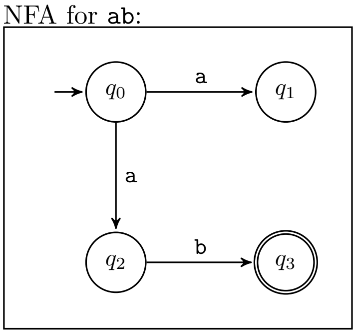
NFA for $(\texttt{ab})^*$:



NFA for $\texttt{a}^*$:



NFA for ab:



NFA for $((\texttt{ab})^*|\texttt{a}^*)$: