Theory of Computer Science

G. Röger
F. Pommerening
Spring Term 2020

University of Basel
Computer Science

Exercise Sheet 12

Due: Wednesday, May 20, 2020

Exercise 12.1 (Polynomial Reduction; 3 marks)
Consider the following decision problems:

HittingSet:

- *Given:* finite set M, set of sets $\mathcal{S} = \{S_1, \dots, S_n\}$ with $S_i \subseteq M$ for all $i \in \{1, \dots, n\}$, natural number $k \in \mathbb{N}_0$
- *Question:* Is there a set H with at most k elements, which contains at least one element from each set in $\mathcal{S}$.

  Formally: Is there a set H with $|H| \leq k$ and $H \cap S_i \neq \emptyset$ for all $i \in \{1, \dots, n\}$?

VertexCover:

- *Given:* undirected graph $G = \langle V, E \rangle$, natural number $k \in \mathbb{N}_0$
- *Question:* Does G have a vertex cover of size at most k, i.e., a set of vertices $C \subseteq V$ with $|C| \leq k$ and $\{u, v\} \cap C \neq \emptyset$ for all $\{u, v\} \in E$?

Prove that HittingSet is NP-hard. You may use that VertexCover is NP-complete.

Exercise 12.2 (Polynomial Reduction; 4 marks)
Consider the following decision problems:

IndSet:

- *Given:* undirected graph $G = \langle V, E \rangle$, natural number $k \in \mathbb{N}_0$
- *Question:* Does G contain an independent set of size k or larger, i.e., is there a set $I \subseteq V$ with $|I| \geq k$ and $\{u, v\} \notin E$ for all $u, v \in I$?

Hence, if there is such a hitting set of size at most k, there is a vertex cover of the required size.
SetPacking:

- *Given:* finite set M, set of sets $\mathcal{S} = \{S_1, \dots, S_n\}$ with $S_i \subseteq M$ for all $i \in \{1, \dots, n\}$, natural number $k \in \mathbb{N}_0$
- *Question:* Is there a set $\mathcal{S}' \subseteq \mathcal{S}$ with $|\mathcal{S}'| \geq k$, such that all sets in $\mathcal{S}'$ are pairwise disjoint, i.e., for all $S_i, S_j \in \mathcal{S}'$ with $S_i \neq S_j$ it holds that $S_i \cap S_j = \emptyset$?

Prove that SetPacking is NP-hard. You may use that IndSet is NP-complete.

Exercise 12.3 (Decidability and NP; 3 marks)
Prove that no undecidable language can be in NP.