Theory of Computer Science

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Exercise Sheet 11

Due: Wednesday, May 13, 2020

Exercise 11.1 (Undecidability; 1+3+2 marks)

(a) Show that for every alphabet Σ there is a context-free grammar $G_{\text{palindrome}}(\Sigma)$ that generates exactly the non-empty palindromes over Σ.

(b) Consider a PCP-instance $\mathcal{I} = \langle (x_1, y_1), \dots, (x_k, y_k) \rangle$ with $x_i, y_i \in \Sigma^+$. Let $\Sigma' = \Sigma \cup \{\#\}$ where $\#$ is a symbol that does not occur in Σ. Specify a context-free grammar $G_{\mathcal{I}}$ over Σ', that generates the following language (with $^{-1}$ we denote the reverse of a word here):

$$\mathcal{L}(G_{\mathcal{I}}) = \{x_{i_1} \dots x_{i_n} \# y_{i_n}^{-1} \dots y_{i_1}^{-1} \mid n \geq 1 \text{ and } i_1, \dots, i_n \in \{1, \dots, k\}\}$$

Moreover, show that $\mathcal{I}$ has a solution iff $\mathcal{L}(G_{\mathcal{I}})$ contains a palindrome.

(c) Use the results from parts (a) and (b) to prove that the intersection problem of *context-free* languages is undecidable.

$$\text{INTERSECTION}_{\text{CF}} : \text{Given two context-free grammars } G_1 \text{ and } G_2, \text{ is } \mathcal{L}(G_1) \cap \mathcal{L}(G_2) = \emptyset?$$

Note: of course you can use the statements of parts (a) and (b) even if you did not solve the exercises.

Exercise 11.2 (Non-deterministic Algorithms; 2+2 marks)

Specify a non-deterministic polynomial algorithm for each of the following problems. This shows that the problems are in NP, a complexity class we will introduce next week.

(a) HittingSet:

- *Given:* finite set M, set of sets $\mathcal{S} = \{S_1, \dots, S_n\}$ with $S_i \subseteq M$ for all $i \in \{1, \dots, n\}$, natural number $k \in \mathbb{N}_0$
- *Question:* Is there a set H with at most k elements, which contains at least one element from each set in $\mathcal{S}$.

  Formally: Is there a set H with $|H| \leq k$ and $H \cap S_i \neq \emptyset$ for all $i \in \{1, \dots, n\}$?

(b) SetPacking:

- *Given:* finite set M, set of sets $\mathcal{S} = \{S_1, \dots, S_n\}$ with $S_i \subseteq M$ for all $i \in \{1, \dots, n\}$, natural number $k \in \mathbb{N}_0$
- *Question:* Is there a set $\mathcal{S}' \subseteq \mathcal{S}$ with $|\mathcal{S}'| \geq k$, such that all sets in $\mathcal{S}'$ are pairwise disjoint, i.e., for all $S_i, S_j \in \mathcal{S}'$ with $S_i \neq S_j$ it holds that $S_i \cap S_j = \emptyset$?