Theory of Computer Science

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Exercise Sheet 10

Due: Wednesday, May 6, 2020

Exercise 10.1 (Transitivity of Reductions; 2 marks)
Show for any languages A, B and C: if $A \leq B$ and $B \leq C$, then $A \leq C$.

Exercise 10.2 (Undecidability; 3 marks)
In each part, give an example of a language L_i with the given properties (without justification), or explain why no such language exists (with a short explanation).

(a) L_1 is undecidable and L_1 and $\overline{L_1}$ are semi-decidable.

(b) L_2 is a type-0 language and decidable.

(c) L_3 is a type-0 language and undecidable.

This question was part of an exam question in 2018.

Exercise 10.3 (Rice's Theorem; 2 marks)
For which of the following languages does Rice's theorem show that the language is undecidable? For each language where Rice's theorem can be used, specify the subset of Turing-computable functions $\mathcal{S}$ for which you use the theorem.
Hint: You do not have to write down any proofs. If Rice's theorem is applicable, specify the set $\mathcal{S}$, otherwise give a short reason (1 sentence) why Rice's theorem is not applicable.

(a) $L = \{w \in \{0,1\}^* \mid M_w \text{ does not terminate with a valid output for any input } \}$

(b) $L = \{w \in \{0,1\}^* \mid M_w \text{ computes the successor function or the predecessor function } \}$

(c) $L = \{w \in \{0,1\}^* \mid M_w \text{ requires an even number of steps on the input } \texttt{0011} \}$

(d) $L = \{w \in \{0,1\}^* \mid \text{No input of } M_w \text{ leads to a valid output containing } \texttt{0} \}$

Exercise 10.4 (Undecidable Grammar Problems; 1.5+1.5 marks)
The *emptiness problem*, the *equivalence problem* and the *intersection problem* for general grammars are defined as:

- EMPTINESS: Given a general grammar G, is $\mathcal{L}(G) = \emptyset$?
- EQUIVALENCE: Given two general grammars G_1 and G_2, is $\mathcal{L}(G_1) = \mathcal{L}(G_2)$?
- INTERSECTION: Given two general grammars G_1 and G_2, is $\mathcal{L}(G_1) \cap \mathcal{L}(G_2) = \emptyset$?

You can use without proof that EMPTINESS is undecidable. (As a bonus exercise, you can also prove this statement with Rice's theorem.)

(a) Show that EQUIVALENCE is undecidable, by reducing EMPTINESS to it.

(b) Show that INTERSECTION is undecidable, by reducing EMPTINESS to it.