Theory of Computer Science

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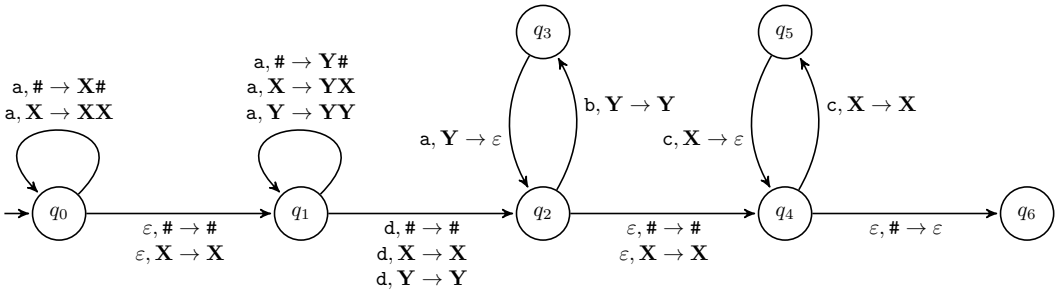
Exercise Sheet 7

Due: Wednesday, April 15, 2020

Exercise 7.1 (Push-down Automata; 2+2 marks)

Consider the push-down automaton (PDA) $M = \langle Q, \Sigma, \Gamma, \delta, q_0, \#\rangle$ with $Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}$, $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}\}$, $\Gamma = \{\mathbf{X}, \mathbf{Y}, \#\}$, and the following transition function δ:



(a) Show that the automaton accepts the word *aaadbabacc* by specifying an accepting sequence of configurations.

(b) What language does this automaton accept?

Exercise 7.2 (Push-down Automata; 2 marks)

Consider language $L = \{\mathtt{a}^i\mathtt{b}^j\mathtt{c}^k \mid i, j, k \geq 0, j = i + k\}$. Specify a push-down automaton (PDA) that accepts L. Remember to specify a complete description with all components. You can specify the transition function δ as a transition diagram.
Note: This exercise was part of the exam in 2018 and gave 5 out of 80 marks there.

Exercise 7.3 (Turing machines; 2 marks)

A *deterministic* Turing machine $M = \langle Q, \Sigma, \Gamma, \delta, q_0, \Box, E\rangle$ is defined like a non-deterministic Turing machine, but with $\delta : (Q \setminus E) \times \Gamma \to Q \times \Gamma \times \{\mathrm{L}, \mathrm{R}, \mathrm{N}\}$. This means that for every non-end state and every symbol from Γ there is exactly one transition (specifying the change of the state and on the tape and the movement of the head).

A Turing machine M that accepts the language

$$L = \{w \in \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}^+ \mid w \text{ contains the same number of } \mathtt{a}, \mathtt{b}, \text{ and } \mathtt{c}\}$$

works with the following loop:

M looks for an `a` on the tape and replaces it with an `x`. Then it searches a `b` on the tape and replaces it with an `x`. Then is searches a `c` on the tape and replaces it with an `x`. If any symbol is not found in this way, M enters an endless loop. Otherwise, it checks if all symbols on the tape have been replaced with `x`. If this is the case, it moves to an accepting state, otherwise the loop starts again.

Specify a transition diagram for a *deterministic* Turing machine with this behavior. Explain your solution by describing which part of the Turing machine is responsible for which part of the loop.