Theory of Computer Science

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Exercise Sheet 6

Due: Wednesday, April 8, 2020

Exercise 6.1 (Regular Expressions and Pumping Lemma for Regular Languages; 3 marks)

Are the following languages over $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ regular? If so, prove it by specifying a regular expression which describes the language. If not, prove it with help of the Pumping-Lemma.

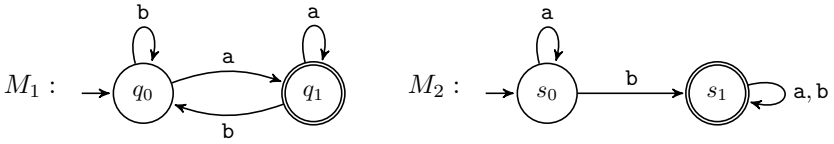
(a) $L_1 = \{\mathtt{a}^n\mathtt{b}^m\mathtt{c}^{n+m} \mid n, m \in \mathbb{N}_0\}$

(b) $L_2 = \{\mathtt{a}^2\mathtt{b}^n\mathtt{a}^2\mathtt{c}^m \mid n, m \in \mathbb{N}_0\}$

Note on marking: One of the languages is regular, the other one is not. The subtask with the regular language is worth 0.5 points, the other subtask 2.5 points.

Exercise 6.2 (Product Automaton; 2 marks)

Given the following DFAs M_1 and M_2.



Specify the product automaton that accepts $\mathcal{L}(M_1) \cap \mathcal{L}(M_2)$.

Exercise 6.3 (Chomsky Normal Form, 3 marks)

Specify a grammar G' in Chomsky normal form that generates the same language as the context-free grammar $G = \langle \Sigma, V, P, \mathrm{S} \rangle$ with $\Sigma = \{\mathtt{a}, \mathtt{b}\}$, $V = \{\mathrm{S}, \mathrm{W}, \mathrm{X}, \mathrm{Y}, \mathrm{Z}\}$ and the following rules in P:

$\mathrm{S} \to \varepsilon$	$\mathrm{S} \to \mathrm{XW}$	$\mathrm{S} \to \mathrm{Z}$	$\mathrm{W} \to \mathrm{X}$	$\mathrm{X} \to \mathtt{aZb}$
$\mathrm{Y} \to \mathrm{W}$	$\mathrm{Y} \to \mathtt{b}\mathrm{Y}$	$\mathrm{Z} \to \mathtt{bb}$	$\mathrm{Z} \to \mathrm{Z}\mathtt{a}$	$\mathrm{X} \to \mathrm{Y}$

Give sufficient intermediate steps.

Exercise 6.4 (Length of Derivations in Chomsky Normal Form; 2 marks)

Let G be a grammar in Chomsky normal form and $w \in \mathcal{L}(G)$ a non-empty word ($w \neq \varepsilon$), which is generated by G. Show that every derivation of w from the start variable of G consists of exactly $2|w| - 1$ steps.