Theory of Computer Science

G. Röger
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University of Basel
Computer Science

Exercise Sheet 4

Due: Wednesday, March 25, 2020

Exercise 4.1 (Predicate logic, 2 + 2 marks)

(a) Show for arbitrary predicate logic formulas φ and ψ that

$$(\forall x \varphi \lor \forall x \psi) \models \forall x(\varphi \lor \psi).$$

(b) Show that in general it is *not* the case that

$$(\forall x \varphi \lor \forall x \psi) \equiv \forall x(\varphi \lor \psi)$$

Specify a counter example with the following signature:$\mathcal{S} = \langle \{x\}, \{\}, \{\}, \{P, Q\} \rangle$, where $ar(P) = ar(Q) = 1$.

Exercise 4.2 (Predicate logic, 1 mark)
Use equivalence transformations to bring the following formula in negation normal form. For this purpose, move negation symbols inwards by using DeMorgan's rule or equivalence $\neg\forall x\varphi \equiv \exists x \neg\varphi$ and $\neg\exists x\varphi \equiv \forall x \neg\varphi$, or eliminate them with double negation.

$$\varphi = \neg\forall x((P(x) \lor \neg Q(x, \mathrm{c})) \land \exists y(P(x) \rightarrow Q(y, x)))$$

Exercise 4.3 (Formal languages and grammars, 1+3+1 marks)
Consider the following formal language over $\{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$:

$$L = \{\mathtt{a}^n \mathtt{b}^m \mathtt{c}^{2n} \mid n \geq 0, m \geq 0\}$$

(a) Is ε an element of L? Justify your answer.

(b) Specify a *complete description* of a formal grammar G that generates L (i.e., $\mathcal{L}(G) = L$). A formal grammar is a four tuple $G = \langle \Sigma, V, P, S \rangle$, remember to define all components of this tuple.

(c) Which types (in the Chomsky-Hierarchy) is your formal grammar part of? You don't have to prove your answers.