Theory of Computer Science

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Exercise Sheet 3

Due: Wednesday, March 18, 2020

Exercise 3.1 (Equivalences; 1.5+1.5 marks)

To formally prove the correctness of a calculus one needs to show for every rule

$$\frac{\varphi_1, \dots, \varphi_n}{\psi}$$

that $\{\varphi_1, \dots, \varphi_n\} \models \psi$.

(a) Prove the correctness of modus tollens.

(b) Prove the correctness of a new inference rule

$$\frac{(\varphi \lor \chi), (\psi \lor \neg\chi)}{(\varphi \lor \psi)}.$$

Exercise 3.2 (Resolution Calculus; 2 Points)

Consider the following knowledge base

$\mathrm{KB} = \{(A \leftrightarrow \neg D), (\neg A \rightarrow (B \lor C)), ((A \rightarrow E) \land (B \lor C \lor F)), (E \rightarrow (F \rightarrow (B \lor C))), (C \rightarrow G), (G \rightarrow \neg C)\}.$

Use the resolution calculus to show that $\mathrm{KB} \models (B \land \neg C)$.

Note: A proof using resolution consists of three steps (see lecture slides for an example). Use the notation from the lecture slides in particular in the last step, that is, use one line for each derived clause together with the derivation's justification. Schöning also uses this notation for the third step on page 36 (the first part of the example, not the visualization).

Exercise 3.3 (Predicate Logic – Terminology; 2 marks)

Classify the following expressions as *terms*, *ground terms*, *atoms*, *formulas* or *meta language* (statements that are not part of predicate logic itself but statement about the semantics). If for an expression several specifications are correct, please list all of them.

In the expressions a and b are constant symbols, x and y variable symbols, f and g function symbols and P and Q predicate symbols.

(a) $\mathrm{P}(x, y)$

(b) $\mathrm{f}(\mathrm{a}, \mathrm{b})$

(c) $\mathcal{I} \models \mathrm{P}(\mathrm{a}, \mathrm{f}(\mathrm{b}))$

(d) $\mathcal{I}, \alpha \models \mathrm{P}(\mathrm{a}, \mathrm{f}(x))$

(e) $\mathrm{f}(\mathrm{g}(x), \mathrm{b})$

(f) $\mathrm{Q}(x)$ is satisfiable.

(g) $(\exists x \mathrm{P}(x, y) \land \mathrm{Q}(x)) \lor \mathrm{P}(y, x)$

(h) $\forall x (\exists y \mathrm{P}(x, y) \land \mathrm{Q}(x)) \lor \mathrm{P}(x, y)$

(i) $\forall x \forall y (\mathrm{P}(x, y) \land \mathrm{Q}(x) \lor \mathrm{P}(\mathrm{f}(y), x))$

(j) $\mathrm{Q}(x) \lor \mathrm{P}(x, y) \equiv \mathrm{P}(x, y) \lor \mathrm{Q}(x)$

Exercise 3.4 (Predicate Logic; 3 Points)

Consider the following predicate logic formula φ with the signature $\langle \{x\}, \{\mathrm{c}\}, \{\mathrm{f}\}, \{\mathrm{P}\} \rangle$.

$$\varphi = (\exists x\, (\mathrm{P}(x) \land \neg \mathrm{P}(\mathrm{f}(x))) \land \forall x\, \neg(\mathrm{f}(x) = \mathrm{c}))$$

Specify a model $\mathcal{I}$ of φ with $\mathcal{I} = \langle U, \cdot^{\mathcal{I}} \rangle$ and $U = \{u_1, u_2, u_3\}$. Prove that $\mathcal{I} \models \varphi$. Why is no variable assignment α required to specify a model of φ?