Theory of Computer Science

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Exercise Sheet 2

Due: Wednesday, March 11, 2020

Exercise 2.1 (Equivalences; 1.5+1.5 marks)

(a) Transform the following formula into CNF by applying the equivalence rules shown in the lecture. For each step, only apply one equivalence rule and also specify it.

$$\phi = (\neg(A \leftrightarrow \neg B) \rightarrow C)$$

(b) Prove that the following formula is a tautology by showing that $\phi \equiv (A \vee \neg A)$ holds. Use the equivalence rules from the lecture, only apply one rule for each step and specify the applied rule.

$$\phi = (A \vee (\neg(A \wedge \neg(\neg A \wedge C)) \vee (A \wedge B)))$$

Exercise 2.2 (Inference; 1+1+1 marks + 1 bonus mark)

You'll find a Java program on the lecture website that checks proofs formulated in propositional logic. Use this program to prove the following statements. For a statement of the form $\mathrm{WB} \models \varphi$ write a text file containing a derivation that only uses formulas from WB as assumptions and that has φ in its last line. An example for this is contained in the file `proof.txt`.
The program checks $\mathrm{WB} \vdash \varphi$. Since the proof system used by the program is correct, this implies $\mathrm{WB} \models \varphi$.

(a) $\{A, B\} \models ((A \wedge B) \vee C)$

(b) $\{(A \wedge B)\} \models (A \rightarrow (B \vee C))$

(c) $\{((A \vee B) \rightarrow (A \rightarrow C)), A\} \models C$

(d) $\{((C \vee D) \leftrightarrow (A \wedge B)), \neg E, (((A \wedge B) \wedge (C \vee D)) \rightarrow E)\} \models \neg(A \wedge B)$
For this exercise, extend the calculus by a new rule *negation-introduction*:

$$\frac{(\varphi \rightarrow \psi), (\varphi \rightarrow \neg\psi)}{\neg\varphi}$$

(e) *Bonus exercise:* To show that a calculus is correct, we have to prove that all rules are correct. Show the correctness of the rule *negation-introduction*

Note on the submission process: please create one text file for each exercise part which contains the derivation. The program must be able to parse the file and accept the derivation as correct. The new rule (*negation-introduction*) requires a new line in the program. Copy this line on your regular submission. The bonus exercise cannot be solved with the program.

Exercise 2.3 (Refutation Theorem; 2 marks)

Prove the refutation theorem, that is, show for any set of formulas KB and any formula φ that

$$\mathrm{KB} \cup \{\varphi\} \text{ is unsatisfiable if and only if } \mathrm{KB} \models \neg\varphi.$$

Please note that there is another exercise on page 2.

Exercise 2.4 (Refutation Completeness; 2 marks)

Let P be a computer program that takes a set of propositional logic formulas as input and returns whether this set of formulas is unsatisfiable.
How can you use P to decide for a knowledge base KB and a propositional logic formula φ whether

(a) KB is satisfiable?

(b) $\text{KB} \models \varphi$?

(c) KB is a tautology?

(d) KB is falsifiable?