Theory of Computer Science

G. Röger
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University of Basel
Computer Science

Exercise Sheet 1

Due: Wednesday, February 26, 2020

On February 24, we introduce the concept of truth tables in the lecture. Please do *not* use them when solving the exercises on this sheet.

Exercise 1.1 (Structural Induction; 3 marks)

We define two functions over binary trees (as presented in the lecture): $\mathit{height} : \mathcal{B} \to \mathbb{N}_0$ maps a binary tree $B \in \mathcal{B}$ to its height and $\mathit{leaves} : \mathcal{B} \to \mathbb{N}_0$ maps a binary tree $B \in \mathcal{B}$ to the number of its leaves.

- $\mathit{height}(\square) = 0$
- $\mathit{height}((B_L, \bigcirc, B_R)) = \max(\mathit{height}(B_L), \mathit{height}(B_R)) + 1$
- $\mathit{leaves}(\square) = 1$
- $\mathit{leaves}((B_L, \bigcirc, B_R)) = \mathit{leaves}(B_L) + \mathit{leaves}(B_R)$

Prove the following property for all binary trees $B \in \mathcal{B}$ by structural induction:

$$\mathit{leaves}(B) \leq 2^{\mathit{height}(B)}.$$

Exercise 1.2 (Formalization in Propositional Logic; 0.5+0.5+0.5+0.5 marks)

Formalize the following statements as propositional formulas. In order to do so, also define appropriate atomic propositions. Take care to fully parenthesize all formulas.

(a) If it does not rain, it is warm.

(b) If Bob is going for a swim, then he always eats ice cream and it is not raining.

(c) Bob is going for a swim exactly if he eats ice cream and it is warm or does not rain.

(d) Either the sun is shining or it is raining (but not both).

Exercise 1.3 (Semantics of Propositional Logic; 2+2 marks)

Consider formula $\varphi = ((A \land \neg B) \to (\neg A \lor \neg C))$ over $\{A, B, C\}$.

(a) Specify a model $\mathcal{I}$ for φ and prove that $\mathcal{I} \models \varphi$.

(b) Specify an interpretation $\mathcal{I}$ under which ϕ does *not* hold and prove that $\mathcal{I} \not\models \varphi$.

Exercise 1.4 (Semantics of Propositional Logic; 1 marks)

Let φ and ψ be propositional formulas over the same set A of atomic propositions and let $\mathcal{I}$ be an interpretation for A. Prove that $\mathcal{I} \models (\varphi \to \psi)$ iff $\mathcal{I} \not\models \varphi$ or $\mathcal{I} \models \psi$.