

# Foundations of Artificial Intelligence

## 38. Automated Planning: Landmarks

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# Planning Heuristics

We discuss **three basic ideas** for general heuristics:

- Delete Relaxation
- Abstraction
- **Landmarks**  $\rightsquigarrow$  this and next chapter

# Planning Heuristics

We discuss **three basic ideas** for general heuristics:

- Delete Relaxation
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- **Landmarks**  $\rightsquigarrow$  this and next chapter

## Basic Idea: Landmarks

**landmark** = something (e.g., an action) that must be part of **every solution**

Estimate solution costs based on unachieved landmarks.

# Automated Planning: Overview

## Chapter overview: automated planning

- 33. Introduction
- 34. Planning Formalisms
- 35.–36. Planning Heuristics: Delete Relaxation
- 37. Planning Heuristics: Abstraction
- 38.–39. Planning Heuristics: Landmarks
  - 38. Landmarks
  - 39. Landmark Heuristics

# Delete Relaxation

# Landmarks and Delete Relaxation

- In this chapter, we discuss a further technique to compute planning heuristics: **landmarks**.
- We restrict ourselves to **delete-free** planning tasks:
  - For a STRIPS task  $\Pi$ , we compute its delete relaxed task  $\Pi^+$ ,
  - and then apply landmark heuristics on  $\Pi^+$ .
- Hence the objective of our landmark heuristics is to approximate the **optimal delete relaxed heuristic  $h^+$**  as accurately as possible.
- More advanced landmark techniques work directly on general planning tasks.

German: Landmarke

# Delete-Free STRIPS planning tasks

reminder:

## Definition (delete-free STRIPS planning task)

A **delete-free STRIPS planning task** is a 4-tuple  $\Pi^+ = \langle V, I, G, A \rangle$  with the following components:

- $V$ : finite set of **state variables**
- $I \subseteq V$ : the **initial state**
- $G \subseteq V$ : the set of **goals**
- $A$ : finite set of **actions**, where for every  $a \in A$ , we define
  - $pre(a) \subseteq V$ : its **preconditions**
  - $add(a) \subseteq V$ : its **add effects**
  - $cost(a) \in \mathbb{N}_0$ : its **cost**

denoted as  $pre(a) \xrightarrow{cost(a)} add(a)$  (omitting set braces)

# Delete-Free STRIPS Planning Task in Normal Form

A delete-free STRIPS planning task  $\langle V, I, G, A \rangle$   
is in **normal form** if

- $I$  consists of exactly one element  $i$ :  $I = \{i\}$
- $G$  consists of exactly one element  $g$ :  $G = \{g\}$
- Every action has at least one precondition.

**German:** Normalform

Every task can easily be transformed  
into an equivalent task in normal form. (**How?**)

- In the following, we assume tasks in normal form.
- Describing  $A$  suffices to describe overall task:
  - $V$  are the variables mentioned in  $A$ 's actions.
  - always  $I = \{i\}$  and  $G = \{g\}$
- In the following, we only describe  $A$ .



# Example: Delete-Free Planning Task in Normal Form

## Example

actions:

- $a_1 = i \xrightarrow{3} x, y$
- $a_2 = i \xrightarrow{4} x, z$
- $a_3 = i \xrightarrow{5} y, z$
- $a_4 = x, y, z \xrightarrow{0} g$

optimal solution?

# Example: Delete-Free Planning Task in Normal Form

## Example

actions:

- $a_1 = i \xrightarrow{3} x, y$
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optimal solution to reach  $\{g\}$  from  $\{i\}$ :

- **plan:**  $a_1, a_2, a_4$
- **cost:**  $3 + 4 + 0 = 7$  ( $= h^+(\{i\})$  because plan is **optimal**)

# Landmarks

# Landmarks

## Definition (landmark)

A **landmark** of a planning task  $\Pi$  is a set of actions  $L$  such that **every plan** must contain an action from  $L$ .

The **cost** of a landmark  $L$ ,  $cost(L)$  is defined as  $\min_{a \in L} cost(a)$ .

↪ landmark cost corresponds to (very simple) admissible heuristic

- Speaking more strictly, landmarks as considered in this course are called **disjunctive action landmarks**.
- other kinds of landmarks exist (fact landmarks, formula landmarks, ...)

**German:** disjunctive Aktionslandmarke, Faktlandmarke, Formellandmarke

# Example: Landmarks

## Example

actions:

- $a_1 = i \xrightarrow{3} x, y$
- $a_2 = i \xrightarrow{4} x, z$
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landmark examples?

# Example: Landmarks

## Example

actions:

- $a_1 = i \xrightarrow{3} x, y$
- $a_2 = i \xrightarrow{4} x, z$
- $a_3 = i \xrightarrow{5} y, z$
- $a_4 = x, y, z \xrightarrow{0} g$

some landmarks:

- $A = \{a_4\}$  (cost 0)
- $B = \{a_1, a_2\}$  (cost 3)
- $C = \{a_1, a_3\}$  (cost 3)
- $D = \{a_2, a_3\}$  (cost 4)
- also:  $\{a_1, a_2, a_3\}$  (cost 3),  $\{a_1, a_2, a_4\}$  (cost 0), ...

# Overview: Landmarks

in the following:

- **exploiting landmarks:**

How can we compute an accurate heuristic for a given set of landmarks?

↪ [this chapter](#)

- **finding landmarks:**

How can we find landmarks?

↪ [next chapter](#)

- **LM-cut heuristic:**

an algorithm to find landmarks and exploit them as heuristic

↪ [next chapter](#)

# Exploiting Landmarks



# Exploiting Landmarks

Assume the set of landmarks  $\mathcal{L} = \{A, B, C, D\}$ .

How to **use**  $\mathcal{L}$  for computing heuristics?

- **sum** the costs:  $0 + 3 + 3 + 4 = 10$   
     $\rightsquigarrow$  **not admissible!**
- **maximize** the costs:  $\max \{0, 3, 3, 4\} = 4$   
     $\rightsquigarrow$  **usually yields a weak heuristic**
- **better:** **hitting sets** or **cost partitioning**

**German:** Hitting-Set, Kostenpartitionierung

# Hitting Sets

## Definition (hitting set)

given: finite support set  $X$ , family of subsets  $\mathcal{F} \subseteq 2^X$ ,  
cost  $c : X \rightarrow \mathbb{R}_0^+$

hitting set:

- subset  $H \subseteq X$  that “hits” all subsets in  $\mathcal{F}$ :  
 $H \cap S \neq \emptyset$  for all  $S \in \mathcal{F}$
- cost of  $H$ :  $\sum_{x \in H} c(x)$

minimum hitting set (MHS):

- hitting set with minimal cost
- “classical” NP-complete problem (Karp, 1972)

# Example: Hitting Sets

## Example

$$X = \{a_1, a_2, a_3, a_4\}$$

$$\mathcal{F} = \{A, B, C, D\}$$

$$\text{with } A = \{a_4\}, B = \{a_1, a_2\}, C = \{a_1, a_3\}, D = \{a_2, a_3\}$$

$$c(a_1) = 3, c(a_2) = 4, c(a_3) = 5, c(a_4) = 0$$

minimum hitting set:

# Example: Hitting Sets

## Example

$$X = \{a_1, a_2, a_3, a_4\}$$

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$$c(a_1) = 3, c(a_2) = 4, c(a_3) = 5, c(a_4) = 0$$

**minimum hitting set:**  $\{a_1, a_2, a_4\}$  with cost  $3 + 4 + 0 = 7$

# Hitting Sets for Landmarks

idea: **landmarks** are interpreted as instance of **minimum hitting set**

## Definition (hitting set heuristic)

Let  $\mathcal{L}$  be a set of landmarks for a delete-free planning task in normal form with actions  $A$ , action costs  $cost$  and initial state  $I$ .

The **hitting set heuristic**  $h^{MHS}(I)$  is defined as the minimal solution cost for the minimum hitting set instance with support set  $A$ , family of subsets  $\mathcal{L}$  and costs  $cost$ .

## Proposition (Hitting Set Heuristic is Admissible)

*The minimum hitting set heuristic  $h^{MHS}$  is admissible.*

Why?

## Approximation of $h^{\text{MHS}}$

- As computing minimal hitting sets is NP-hard, we want to approximate  $h^{\text{MHS}}$  in polynomial time.

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## Optimal Cost Partitioning (Karpas & Domshlak, 2009)

idea: Construct a **linear program** (LP) for  $\mathcal{L}$ .

- **rows** (constraints) correspond to **actions**
- **columns** (variables) correspond to **landmarks**
- **entries**: 1 if row action is contained in column landmark; 0 otherwise
- **objective**: maximize sum of variables

heuristic value  $h^{\text{OCP}}$  (optimal cost partitioning):  
objective value of LP

# Example: Optimal Cost Partitioning

## Example

$$\text{cost}(a_1) = 3, \text{cost}(a_2) = 4, \text{cost}(a_3) = 5, \text{cost}(a_4) = 0$$

$$\mathcal{L} = \{A, B, C, D\}$$

$$\text{with } A = \{a_4\}, B = \{a_1, a_2\}, C = \{a_1, a_3\}, D = \{a_2, a_3\}$$

**LP:** maximize  $a + b + c + d$  subject to  $a, b, c, d \geq 0$  and

$$b + c \leq 3$$

$$b + d \leq 4$$

$$c + d \leq 5$$

$$a \leq 0$$



# Example: Optimal Cost Partitioning

## Example

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$$\begin{array}{rcccccl}
 & & b & + & c & & \leq & 3 & a_1 \\
 & & b & + & & & d & \leq & 4 & a_2 \\
 & & & & c & + & d & \leq & 5 & a_3 \\
 a & & & & & & & \leq & 0 & a_4 \\
 A & B & C & D & & & & & & 
 \end{array}$$

# Example: Optimal Cost Partitioning

## Example

$$\text{cost}(a_1) = 3, \text{cost}(a_2) = 4, \text{cost}(a_3) = 5, \text{cost}(a_4) = 0$$

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**solution: ?**

# Example: Optimal Cost Partitioning

## Example

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**LP:** maximize  $a + b + c + d$  subject to  $a, b, c, d \geq 0$  and

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 A & B & C & D & & & & & & 
 \end{array}$$

**solution:**  $a = 0, b = 1, c = 2, d = 3 \rightsquigarrow h^{\text{OCP}}(I) = 6$

# Relationship of Heuristics

## Proposition ( $h^{\text{OCP}}$ vs. $h^{\text{MHS}}$ )

Let  $\mathcal{L}$  be a set of landmarks for a planning task with initial state  $I$ .

Then  $h^{\text{OCP}}(I) \leq h^{\text{MHS}}(I) \leq h^+(I)$

The heuristic  $h^{\text{OCP}}$  can be computed in polynomial time because linear programs can be solved in polynomial time.

# Summary

# Summary

- **Landmarks** are action sets such that every plan must contain at least one of the actions.
- **Hitting sets** yield the most accurate heuristic for a given set of landmarks, but the computation is NP-hard.
- **Optimal cost partitioning** is a polynomial approach for the computation of informative landmark heuristics.