

Foundations of Artificial Intelligence

11. State-Space Search: Uniform Cost Search

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State-Space Search: Overview

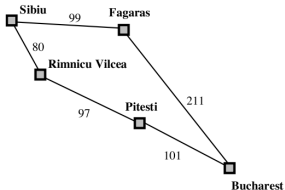
Chapter overview: state-space search

- 5.–7. Foundations
- 8.–12. Basic Algorithms
 - 8. Data Structures for Search Algorithms
 - 9. Tree Search and Graph Search
 - 10. Breadth-first Search
 - 11. Uniform Cost Search
 - 12. Depth-first Search and Iterative Deepening
- 13.–19. Heuristic Algorithms

Introduction

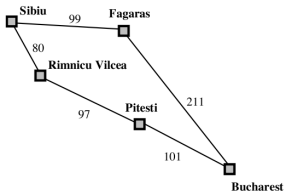
Uniform Cost Search

- breadth-first search optimal if all action costs equal
- otherwise no optimality guarantee \rightsquigarrow [example](#):



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remedy: **uniform cost search**

- always expand a node with **minimal path cost**
($n.path_cost$ a.k.a. $g(n)$)
- **implementation**: **priority queue** (min-heap) for open list

Algorithm

Reminder: Generic Graph Search Algorithm

reminder from Chapter 9:

Generic Graph Search

```
open := new OpenList
open.insert(make_root_node())
closed := new ClosedList
while not open.is_empty():
    n := open.pop()
    if closed.lookup(n.state) = none:
        closed.insert(n)
        if is_goal(n.state):
            return extract_path(n)
        for each  $\langle a, s' \rangle \in \text{succ}(n.\text{state})$ :
            n' := make_node(n, a, s')
            open.insert(n')
return unsolvable
```

Uniform Cost Search

Uniform Cost Search

```
open := new MinHeap ordered by g
open.insert(make_root_node())
closed := new HashSet
while not open.is_empty():
    n := open.pop_min()
    if n.state  $\notin$  closed:
        closed.insert(n)
        if is_goal(n.state):
            return extract_path(n)
        for each  $\langle a, s' \rangle \in$  succ(n.state):
            n' := make_node(n, a, s')
            open.insert(n')
return unsolvable
```


Uniform Cost Search: Discussion

Adapting generic graph search to uniform cost search:

- here, early goal tests/early updates of the closed list **not** a good idea. (Why not?)
- as in BFS-Graph, a **set** is sufficient for the closed list
- a tree search variant is possible, but rare:
has the same disadvantages as BFS-Tree
and in general **not even semi-complete** (Why not?)

Remarks:

- identical to **Dijkstra's algorithm** for shortest paths
- for both: variants with/without delayed duplicate elimination

Uniform Cost Search: Improvements

possible improvements:

- if action costs are small integers, **bucket heaps** often more efficient
- additional early duplicate tests for generated nodes can reduce memory requirements
 - can be beneficial or detrimental for runtime
 - must be careful to keep shorter path to duplicate state

Properties

Completeness and Optimality

properties of uniform cost search:

- uniform cost search is **complete** (Why?)
- uniform cost search is **optimal** (Why?)

Time and Space Complexity

properties of uniform cost search:

- **Time complexity** depends on distribution of action costs (no simple and accurate bounds).
 - Let $\varepsilon := \min_{a \in A} \text{cost}(a)$ and consider the case $\varepsilon > 0$.
 - Let c^* be the optimal solution cost.
 - Let b be the branching factor and consider the case $b \geq 2$.
 - Then the time complexity is at most $O(b^{\lfloor c^*/\varepsilon \rfloor + 1})$. (Why?)
 - often a very weak upper bound
- **space complexity** = time complexity

Summary

Summary

uniform cost search: expand nodes in order of **ascending path costs**

- usually as a graph search
- then corresponds to Dijkstra's algorithm
- **complete** and **optimal**