

# Foundations of Artificial Intelligence

## 5. State-Space Search: State Spaces

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# State-Space Search Problems

# Classical State-Space Search Problems Informally

(Classical) state-space search problems are among the “simplest” and most important classes of AI problems.

objective of the agent:

- from a given initial state
- apply a sequence of actions
- in order to reach a goal state

performance measure: minimize total action cost

# Motivating Example: 15-Puzzle

9	2	12	6
5	7	14	13
3		1	11
15	4	10	8



1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

# Classical Assumptions

“classical” assumptions:

- no other agents in the environment (**single-agent**)
- always knows state of the world (**fully observable**)
- state only changed by the agent (**static**)
- finite number of states/actions (in particular **discrete**)
- actions have **deterministic** effect on the state

↪ can all be generalized (but not in this part of the course)

For simplicity, we omit “**classical**” in the following.

# Classification

Classification:

## State-Space Search

environment:

- **static** vs. dynamic
- **deterministic** vs. non-deterministic vs. stochastic
- **fully** vs. partially vs. not **observable**
- **discrete** vs. continuous
- **single-agent** vs. multi-agent

problem solving method:

- **problem-specific** vs. general vs. learning

# Search Problem Examples

- **toy problems**: combinatorial puzzles (Rubik's Cube, 15-puzzle, towers of Hanoi, ...)
- **scheduling** of events, flights, manufacturing tasks
- **query optimization** in databases
- behavior of **NPCs** in computer games
- **code optimization** in compilers
- **verification** of soft- and hardware
- **sequence alignment** in bioinformatics
- **route planning** (e.g., Google Maps)
- ...

**thousands** of practical examples

# State-Space Search: Overview

## Chapter overview: state-space search

- 5.–7. Foundations
  - 5. State Spaces
  - 6. Representation of State Spaces
  - 7. Examples of State Spaces
- 8.–12. Basic Algorithms
- 13.–19. Heuristic Algorithms



# Formalization

# Formalization

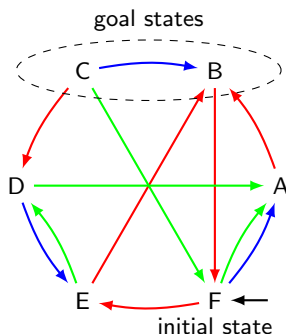
## preliminary remarks:

- to cleanly study search problems we need a **formal model**
- fundamental concept: **state spaces**
- state spaces are (labeled, directed) **graphs**
- **paths** to goal states represent **solutions**
- **shortest paths** correspond to **optimal solutions**

# State Spaces: Example

State spaces are often depicted as **directed graphs**.

- **states**: graph vertices
- **transitions**: labeled arcs  
(here: colors instead of labels)
- **initial state**: incoming arrow
- **goal states**: marked  
(here: by the dashed ellipse)
- **actions**: the arc labels
- **action costs**: described separately  
(or implicitly = 1)



# State Spaces

## Definition (state space)

A **state space** or **transition system** is a 6-tuple

$\mathcal{S} = \langle S, A, cost, T, s_0, S_\star \rangle$  with

- $S$ : finite set of **states**
- $A$ : finite set of **actions**
- $cost : A \rightarrow \mathbb{R}_0^+$  **action costs**
- $T \subseteq S \times A \times S$  **transition relation**; **deterministic** in  $\langle s, a \rangle$   
(see next slide)
- $s_0 \in S$  **initial state**
- $S_\star \subseteq S$  set of **goal states**

**German:** Zustandsraum, Transitionssystem, Zustände, Aktionen, Aktionskosten, Transitions-/Übergangsrelation, deterministisch, Anfangszustand, Zielzustände

# State Spaces: Transitions, Determinism

## Definition (transition, deterministic)

Let  $\mathcal{S} = \langle S, A, cost, T, s_0, S_* \rangle$  be a state space.

The triples  $\langle s, a, s' \rangle \in T$  are called **(state) transitions**.

We say  $\mathcal{S}$  **has the transition**  $\langle s, a, s' \rangle$  if  $\langle s, a, s' \rangle \in T$ .

We write this as  $s \xrightarrow{a} s'$ , or  $s \rightarrow s'$  when  $a$  does not matter.

Transitions are **deterministic** in  $\langle s, a \rangle$ : it is forbidden to have both  $s \xrightarrow{a} s_1$  and  $s \xrightarrow{a} s_2$  with  $s_1 \neq s_2$ .

# State Spaces: Terminology

We use common terminology from graph theory.

## Definition (predecessor, successor, applicable action)

Let  $\mathcal{S} = \langle S, A, cost, T, s_0, S_* \rangle$  be a state space.

Let  $s, s' \in S$  be states with  $s \rightarrow s'$ .

- $s$  is a **predecessor** of  $s'$
- $s'$  is a **successor** of  $s$

If  $s \xrightarrow{a} s'$ , then action  $a$  is **applicable** in  $s$ .

**German:** Vorgänger, Nachfolger, anwendbar

# State Spaces: Terminology

We use common terminology from graph theory.

## Definition (path)

Let  $S = \langle S, A, cost, T, s_0, S_* \rangle$  be a state space.

Let  $s^{(0)}, \dots, s^{(n)} \in S$  be states and  $\pi_1, \dots, \pi_n \in A$  be actions such that  $s^{(0)} \xrightarrow{\pi_1} s^{(1)}, \dots, s^{(n-1)} \xrightarrow{\pi_n} s^{(n)}$ .

- $\pi = \langle \pi_1, \dots, \pi_n \rangle$  is a **path** from  $s^{(0)}$  to  $s^{(n)}$
- **length** of  $\pi$ :  $|\pi| = n$
- **cost** of  $\pi$ :  $cost(\pi) = \sum_{i=1}^n cost(\pi_i)$

**German:** Pfad, Länge, Kosten

- paths may have length 0
- sometimes “path” is used for state sequence  $\langle s^{(0)}, \dots, s^{(n)} \rangle$  or sequence  $\langle s^{(0)}, \pi_1, s^{(1)}, \dots, s^{(n-1)}, \pi_n, s^{(n)} \rangle$

# State Spaces: Terminology

more terminology:

## Definition (reachable, solution, optimal)

Let  $\mathcal{S} = \langle S, A, cost, T, s_0, S_\star \rangle$  be a state space.

- state  $s$  is **reachable** if a path from  $s_0$  to  $s$  exists
- paths from  $s \in S$  to some state  $s_\star \in S_\star$  are **solutions for/from  $s$**
- solutions for  $s_0$  are called **solutions for  $\mathcal{S}$**
- **optimal solutions** (for  $s$ ) have minimal costs among all solutions (for  $s$ )

**German:** erreichbar, Lösung von/für  $s$ , optimale Lösung



# State-Space Search

# State-Space Search

## State-Space Search

**State-space search** is the algorithmic problem of finding solutions in state spaces or proving that no solution exists.

In **optimal** state-space search, only optimal solutions may be returned.

**German:** Zustandsraumsuche, optimale Zustandsraumsuche

# Learning Objectives for State-Space Search

## Learning Objectives for the Topic of State-Space Search

- **understanding state-space search:**  
What is the problem and how can we formalize it?
- **evaluate search algorithms:**  
completeness, optimality, time/space complexity
- **get to know search algorithms:**  
uninformed vs. informed; tree and graph search
- **evaluate heuristics for search algorithms:**  
goal-awareness, safety, admissibility, consistency
- **efficient implementation** of search algorithms
- **experimental evaluation** of search algorithms
- **design and comparison of heuristics** for search algorithms

# Summary

# Summary

- **classical state-space search problems:** find action sequence from initial state to a goal state
- **performance measure:** sum of action costs
- formalization via **state spaces:**
  - **states, actions, action costs, transitions, initial state, goal states**
- terminology for transitions, paths, solutions
- definition of (optimal) state-space search