Foundations of Artificial Intelligence

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Spring Term 2020

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Exercise Sheet 7

Due: April 15, 2020

Important: For submission, consult the rules at the end of the exercise. Non-adherence to the rules will lead to your submission not being corrected.

Exercise 7.1 (2+0.5+0.5 marks)

Consider the propositional formula $\varphi = (a \vee b) \wedge (\neg b \vee c) \wedge (\neg a \vee \neg b)$.

(a) Formalize φ as a binary constraint network.

(b) Provide a consistent partial variable assignment.

(c) Provide an inconsistent partial variable assignment.

Exercise 7.2 (1+1+1 marks)

Consider the constraint network $\mathcal{C} = \langle \{a, b, c\}, \mathrm{dom}, (R_{uv}) \rangle$ with $\mathrm{dom}(x) = \{0, 1\}$ for $x \in V$, $R_{ab} = R_{bc} = \{\langle 0, 0 \rangle, \langle 0, 1 \rangle, \langle 1, 1 \rangle\}$ and $R_{ac} = \{\langle 0, 1 \rangle\}$.

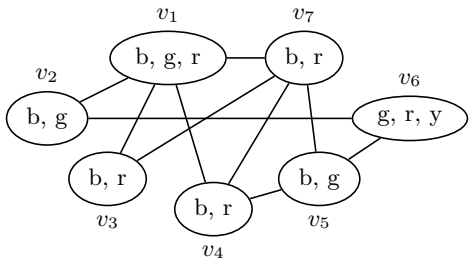
(a) Define a constraint network $\mathcal{C}^1$ with $\mathcal{C}^1 \sqsubset \mathcal{C}$ and $\mathcal{C}^1 \equiv \mathcal{C}$.

(b) Define a constraint network $\mathcal{C}^2$ with $\mathcal{C}^2 \sqsubset \mathcal{C}$ and $\mathcal{C}^2 \not\equiv \mathcal{C}$.

(c) Define a constraint network $\mathcal{C}^3$ with $\mathcal{C}^3 \not\sqsubseteq \mathcal{C}$, $\mathcal{C} \not\sqsubseteq \mathcal{C}^3$ and $\mathcal{C} \equiv \mathcal{C}^3$.

Exercise 7.3 (1+2 marks)

Consider the constraint network for the graph coloring problem that has been introduced on slide 10 of chapter 24 in the print version of the lecture slides:



Use the following static strategies on variable and value orderings:

- Variable ordering:
  - select a variable according to the *minimum remaining values* variable ordering criterion;
  - if there is more than one such variable, break ties according to the *most constraining variable* variable ordering criterion;
  - if the choice is still not unique, break ties by selecting the variable with the smallest index.
- Value ordering: reverse alphabetical

(a) Provide the variable and value orderings induced by the static ordering strategies.

(b) Provide the search tree that is created by applying naive backtracking on the depicted problem using the static variable and value orderings from the first part of the exercise. Depict the search tree in a similar style as in the lecture slides.

Exercise 7.4 (1+2 marks)

Consider a constraint network $\mathcal{C}$ denoting a Latin square of size 3 with the partial assignment $\alpha = \{\text{A1} \mapsto 1, \text{B1} \mapsto 2, \text{A3} \mapsto 3\}$:

	1	2	3
A	1		3
B	2		
C			

In the following, you may assume that the already filled out squares are fixed, i.e., that the domain of the corresponding variables contains only the single entry that encodes the depicted value. The domain of the remaining variables contains all 3 possible values, which leads to the following domains for all variables:

$$\begin{array}{lll} \text{dom}(\text{A1}) = \{1\} & \text{dom}(\text{A2}) = \{1,2,3\} & \text{dom}(\text{A3}) = \{3\} \\ \text{dom}(\text{B1}) = \{2\} & \text{dom}(\text{B2}) = \{1,2,3\} & \text{dom}(\text{B3}) = \{1,2,3\} \\ \text{dom}(\text{C1}) = \{1,2,3\} & \text{dom}(\text{C2}) = \{1,2,3\} & \text{dom}(\text{C3}) = \{1,2,3\} \end{array}$$

(a) Determine the domains of all variables after applying forward checking in α.

(b) Apply the AC-3 algorithm that has been presented on slide 21 of chapter 25 in the print version of the lecture slides on the constraint network $\mathcal{C}$ with the domains that are the result of (a) until arc consistency is enforced. Select the variables u and v in each iteration of the while loop such that the domain of u changes in the call to `revise`$(\mathcal{C}, u, v)$. Provide u, v, and $\text{dom}(u)$ in each iteration. Note that you do *not* have to provide the elements that are inserted into the queue, and you may stop the algorithm as soon as there are no variables u and v such that $\text{dom}(u)$ changes.

Submission rules:

- Upload a single PDF file (ending .pdf). If you want to submit handwritten parts, include their scans in the single PDF. Put the names of all group members on top of the first page. Use page numbers or put your names on each page. Make sure your PDF has size A4 (fits the page size if printed on A4).
- Only upload one submission per group. Do not upload several versions, i.e., if you need to resubmit, use the same file name again so that the previous submission is overwritten.