

# Theory of Computer Science

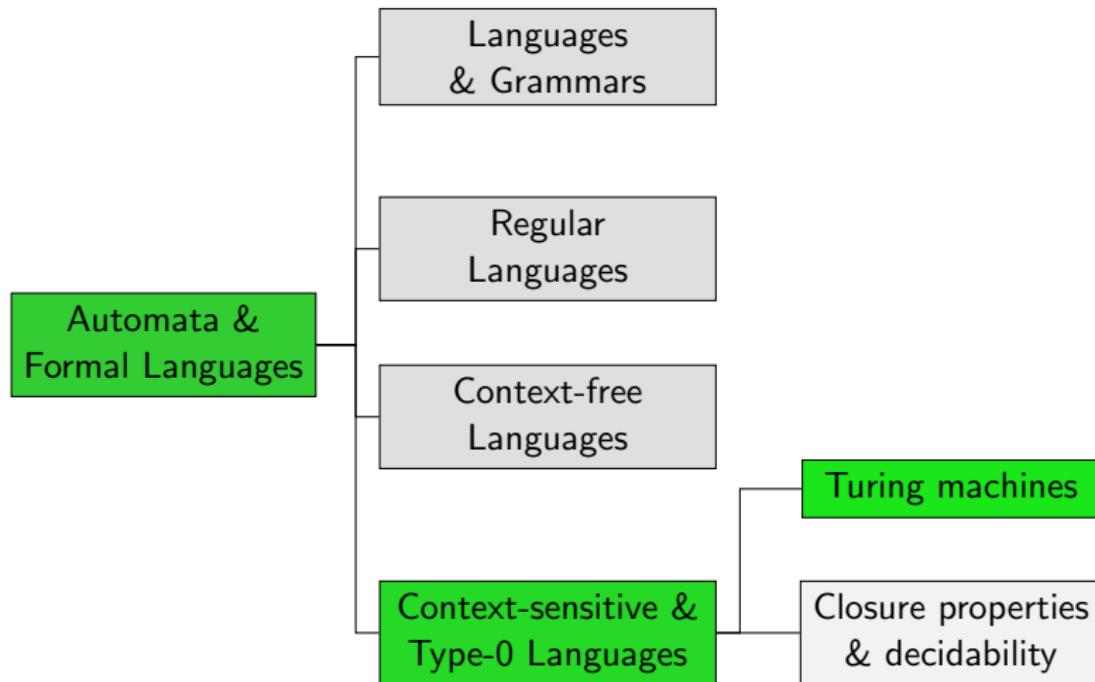
## C8. Type-1 and Type-0 Languages: Closure & Decidability

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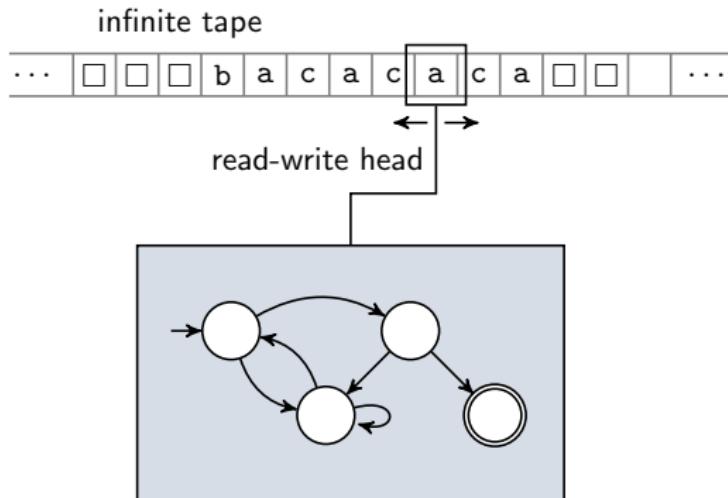
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# Overview



# Turing Machines vs. Grammars

# Reminder: Turing Machines – Conceptually



# Reminder: Nondeterministic Turing Machine

## Definition (Nondeterministic Turing Machine)

A nondeterministic Turing machine (NTM) is given by a 7-tuple  $M = \langle Q, \Sigma, \Gamma, \delta, q_0, \square, E \rangle$  with:

- $Q$  finite non-empty set of **states**
- $\Sigma \neq \emptyset$  finite **input alphabet**
- $\Gamma \supset \Sigma$  finite **tape alphabet**
- $\delta : (Q \setminus E) \times \Gamma \rightarrow \mathcal{P}(Q \times \Gamma \times \{L, R, N\})$  **transition function**
- $q_0 \in Q$  **start state**
- $\square \in \Gamma \setminus \Sigma$  **blank symbol**
- $E \subseteq Q$  **end states**

# One Automata Model for Two Grammar Types?

Don't we need  
different automata models for  
context-sensitive and type-0  
languages?



## Linear Bounded Automata: Idea

- Linear bounded automata are NTMs that may only use the part of the tape occupied by the input word.
- one way of formalizing this: NTMs where blank symbol may never be replaced by a different symbol

# Linear Bounded Turing Machines: Definition

## Definition (Linear Bounded Automata)

An NTM  $M = \langle Q, \Sigma, \Gamma, \delta, q_0, \square, E \rangle$

is called a **linear bounded automaton (LBA)**

if for all  $q \in Q \setminus E$  and all transition rules  $\langle q', c, y \rangle \in \delta(q, \square)$   
we have  $c = \square$ .

German: linear beschränkte Turingmaschine

# LBAs Accept Type-1 Languages

## Theorem

*The languages that can be accepted by linear bounded automata are exactly the context-sensitive (type-1) languages.*

Without proof.

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proof sketch for grammar  $\Rightarrow$  NTM direction:

- computation of the NTM follows the production of the word in the grammar **in opposite order**
- accept when only start symbol (and blanks) are left on the tape
- because language is context-sensitive, we never need additional space on the tape (empty word needs special treatment)

# NTMs Accept Type-0 Languages

## Theorem

*The languages that can be accepted by nondeterministic Turing machines are exactly the type-0 languages.*

Without proof.

# NTMs Accept Type-0 Languages

## Theorem

*The languages that can be accepted by nondeterministic Turing machines are exactly the type-0 languages.*

Without proof.

proof sketch for grammar  $\Rightarrow$  NTM direction:

- analogous to previous proof
- for grammar rules  $w_1 \rightarrow w_2$  with  $|w_1| > |w_2|$ , we must “insert” symbols into the existing tape content; this is a bit tedious, but not very difficult

# Deterministic Turing Machines

## Definition (Deterministic Turing Machine)

A **deterministic Turing machine (DTM)** is a Turing machine  
 $M = \langle Q, \Sigma, \Gamma, \delta, q_0, \square, E \rangle$  with  
 $\delta : (Q \setminus E) \times \Gamma \rightarrow Q \times \Gamma \times \{L, R, N\}.$

German: deterministische Turingmaschine

# Deterministic Turing Machines vs. Type-0 Languages

## Theorem

*For every type-0 language  $L$  there is a deterministic Turing machine  $M$  with  $\mathcal{L}(M) = L$ .*

Without proof.

# Deterministic Turing Machines vs. Type-0 Languages

## Theorem

*For every type-0 language  $L$  there is a deterministic Turing machine  $M$  with  $\mathcal{L}(M) = L$ .*

Without proof.

proof sketch:

- Let  $M'$  be an NTM with  $\mathcal{L}(M') = L$ .
- It is possible to construct a DTM that systematically searches for an accepting configuration in the computation tree of  $M'$ .

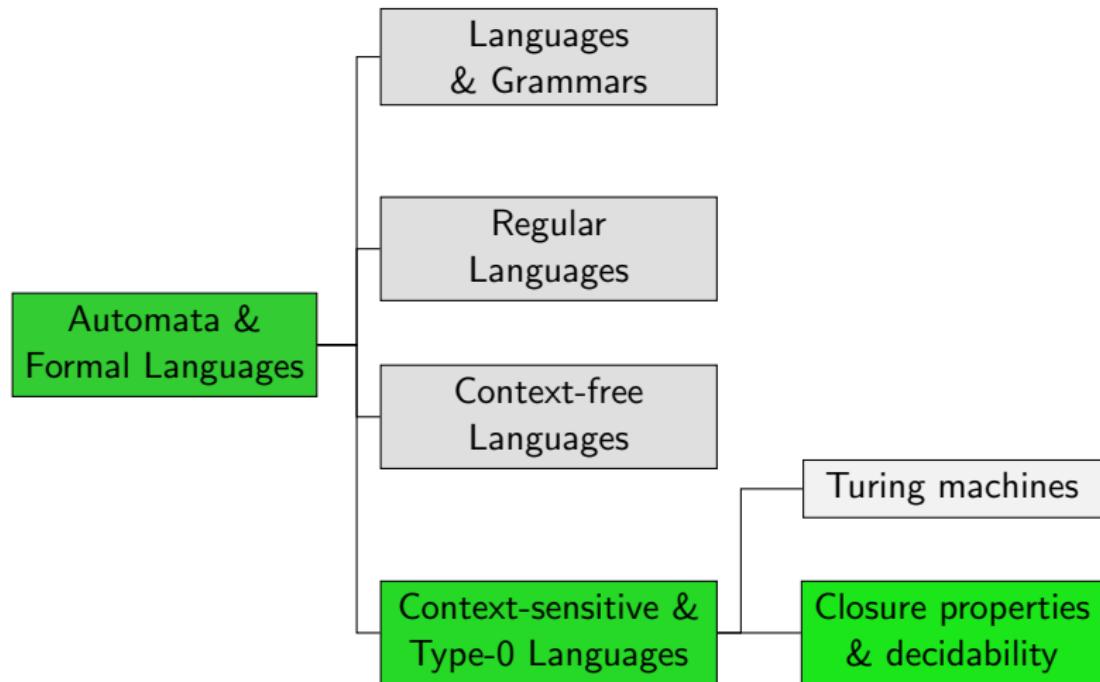
Note: It is an open problem whether an analogous theorem holds for type-1 languages and deterministic LBAs.

# Questions



# Closure Properties and Decidability

# Overview



# Closure Properties

	Intersection	Union	Complement	Product	Star
Type 3	Yes	Yes	Yes	Yes	Yes
Type 2	No	Yes	No	Yes	Yes
Type 1	Yes <sup>(2)</sup>	Yes <sup>(1)</sup>	Yes <sup>(2)</sup>	Yes <sup>(1)</sup>	Yes <sup>(1)</sup>
Type 0	Yes <sup>(2)</sup>	Yes <sup>(1)</sup>	No <sup>(3)</sup>	Yes <sup>(1)</sup>	Yes <sup>(1)</sup>

Proofs?

- (1) proof via grammars, similar to context-free cases
- (2) without proof
- (3) proof in later chapters (part D)

# Decidability

	Word problem	Emptiness problem	Equivalence problem	Intersection problem
Type 3	Yes	Yes	Yes	Yes
Type 2	Yes	Yes	No	No
Type 1	Yes <sup>(1)</sup>	No <sup>(3)</sup>	No <sup>(2)</sup>	No <sup>(2)</sup>
Type 0	No <sup>(4)</sup>	No <sup>(4)</sup>	No <sup>(4)</sup>	No <sup>(4)</sup>

## Proofs?

- (1) same argument we used for context-free languages
- (2) because already undecidable for context-free languages
- (3) without proof
- (4) proofs in later chapters (part D)

# Questions



# Summary

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- Turing machines accept exactly the type-0 languages.  
This is also true for deterministic Turing machines.
- Linear bounded automata accept exactly the context-sensitive languages.
- The context-sensitive and type-0 languages are closed under almost all usual operations.
  - exception: type-0 not closed under complement
- For context-sensitive and type-0 languages almost no problem is decidable.
  - exception: word problem for context-sensitive lang. decidable

# What's Next?

contents of this course:

- A. **background ✓**
  - ▷ mathematical foundations and proof techniques
- B. **logic ✓**
  - ▷ How can knowledge be represented?
  - How can reasoning be automated?
- C. **automata theory and formal languages**
  - ▷ What is a computation?
- D. **Turing computability**
  - ▷ What can be computed at all?
- E. **complexity theory**
  - ▷ What can be computed efficiently?
- F. **more computability theory**
  - ▷ Other models of computability

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