Theory of Computer Science

C8. Type-1 and Type-0 Languages: Closure & Decidability

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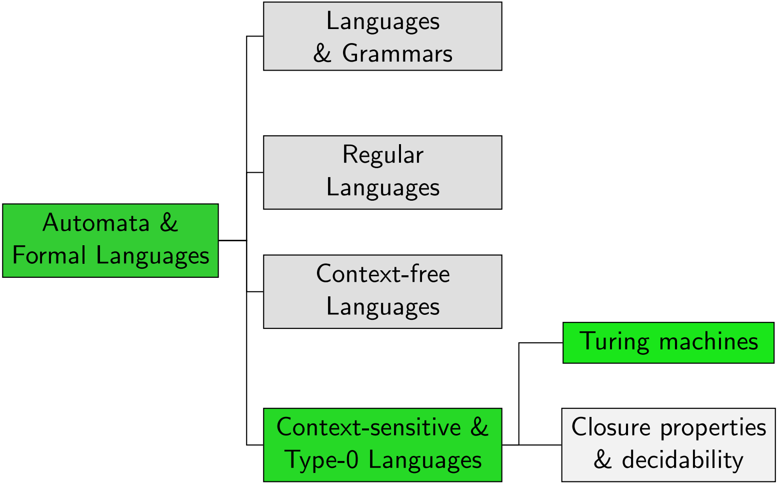
April 8, 2019

C8.1 Turing Machines vs. Grammars

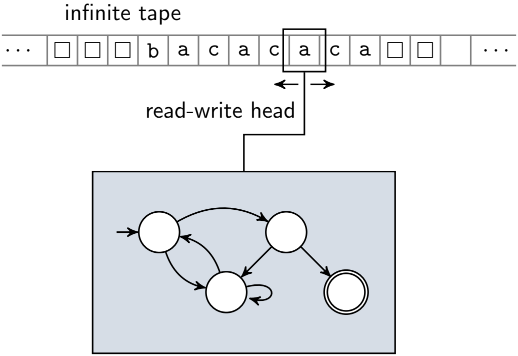
C8.2 Closure Properties and Decidability

Overview


Languages & Grammars
Regular Languages
Automata & Formal Languages
Context-free Languages
Turing machines
Context-sensitive & Type-0 Languages
Closure properties & decidability

C8.1 Turing Machines vs. Grammars

Reminder: Turing Machines – Conceptually



Reminder: Nondeterministic Turing Machine

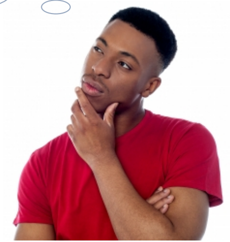
Definition (Nondeterministic Turing Machine)

A nondeterministic Turing machine (NTM) is given by a 7-tuple $M = \langle Q, \Sigma, \Gamma, \delta, q_0, \square, E \rangle$ with:

- Q finite non-empty set of states
- $\Sigma \neq \emptyset$ finite input alphabet
- $\Gamma \supset \Sigma$ finite tape alphabet
- $\delta : (Q \setminus E) \times \Gamma \to \mathcal{P}(Q \times \Gamma \times \{\mathrm{L}, \mathrm{R}, \mathrm{N}\})$ transition function
- $q_0 \in Q$ start state
- $\square \in \Gamma \setminus \Sigma$ blank symbol
- $E \subseteq Q$ end states

One Automata Model for Two Grammar Types?

Don't we need
different automata models for
context-sensitive and type-0
languages?



Picture courtesy of stockimages / FreeDigitalPhotos.net

Linear Bounded Automata: Idea

- Linear bounded automata are NTMs that may only use the part of the tape occupied by the input word.
- one way of formalizing this: NTMs where blank symbol may never be replaced by a different symbol

Linear Bounded Turing Machines: Definition

Definition (Linear Bounded Automata)

An NTM $M = \langle Q, \Sigma, \Gamma, \delta, q_0, \square, E \rangle$
is called a linear bounded automaton (LBA)
if for all $q \in Q \setminus E$ and all transition rules $\langle q', c, y \rangle \in \delta(q, \square)$
we have $c = \square$.

German: linear beschränkte Turingmaschine

LBAs Accept Type-1 Languages

Theorem

The languages that can be accepted by linear bounded automata are exactly the context-sensitive (type-1) languages.

Without proof.

proof sketch for grammar $\Rightarrow$ NTM direction:

- computation of the NTM follows the production of the word in the grammar in opposite order
- accept when only start symbol (and blanks) are left on the tape
- because language is context-sensitive, we never need additional space on the tape (empty word needs special treatment)

NTMs Accept Type-0 Languages

Theorem

The languages that can be accepted by nondeterministic Turing machines are exactly the type-0 languages.

Without proof.

proof sketch for grammar $\Rightarrow$ NTM direction:

- analogous to previous proof
- for grammar rules $w_1 \to w_2$ with $|w_1| > |w_2|$, we must "insert" symbols into the existing tape content; this is a bit tedious, but not very difficult

Deterministic Turing Machines

Definition (Deterministic Turing Machine)

A deterministic Turing machine (DTM) is a Turing machine $M = \langle Q, \Sigma, \Gamma, \delta, q_0, \Box, E \rangle$ with $\delta : (Q \setminus E) \times \Gamma \to Q \times \Gamma \times \{\mathsf{L}, \mathsf{R}, \mathsf{N}\}$.

German: deterministische Turingmaschine

Deterministic Turing Machines vs. Type-0 Languages

Theorem

For every type-0 language L there is a deterministic Turing machine M with $\mathcal{L}(M) = L$.

Without proof.

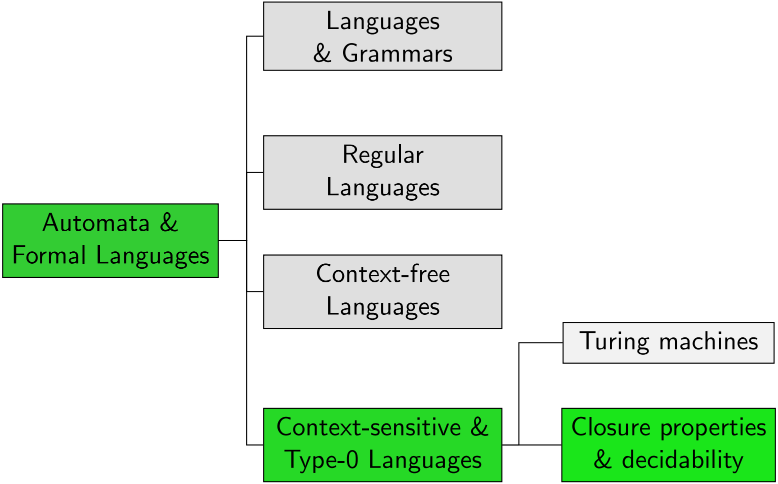
proof sketch:

- Let M' be an NTM with $\mathcal{L}(M') = L$.
- It is possible to construct a DTM that systematically searches for an accepting configuration in the computation tree of M'.

Note: It is an open problem whether an analogous theorem holds for type-1 languages and deterministic LBAs.

C8.2 Closure Properties and Decidability

Overview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Closure Properties

	Intersection	Union	Complement	Product	Star
Type 3	Yes	Yes	Yes	Yes	Yes
Type 2	No	Yes	No	Yes	Yes
Type 1	Yes$^{(2)}$	Yes$^{(1)}$	Yes$^{(2)}$	Yes$^{(1)}$	Yes$^{(1)}$
Type 0	Yes$^{(2)}$	Yes$^{(1)}$	No$^{(3)}$	Yes$^{(1)}$	Yes$^{(1)}$

Proofs?

(1) proof via grammars, similar to context-free cases

(2) without proof

(3) proof in later chapters (part D)

Decidability

	Word problem	Emptiness problem	Equivalence problem	Intersection problem
Type 3	Yes	Yes	Yes	Yes
Type 2	Yes	Yes	No	No
Type 1	Yes(1)	No(3)	No(2)	No(2)
Type 0	No(4)	No(4)	No(4)	No(4)

Proofs?

(1) same argument we used for context-free languages

(2) because already undecidable for context-free languages

(3) without proof

(4) proofs in later chapters (part D)

Summary

- Turing machines accept exactly the type-0 languages. This is also true for deterministic Turing machines.
- Linear bounded automata accept exactly the context-sensitive languages.
- The context-sensitive and type-0 languages are closed under almost all usual operations.
  - exception: type-0 not closed under complement
- For context-sensitive and type-0 languages almost no problem is decidable.
  - exception: word problem for context-sensitive lang. decidable

What's Next?

contents of this course:

A. background ✓
   - mathematical foundations and proof techniques

B. logic ✓
   - How can knowledge be represented?
     How can reasoning be automated?

C. automata theory and formal languages ✓
   - What is a computation?

D. Turing computability
   - What can be computed at all?

E. complexity theory
   - What can be computed efficiently?

F. more computability theory
   - Other models of computability