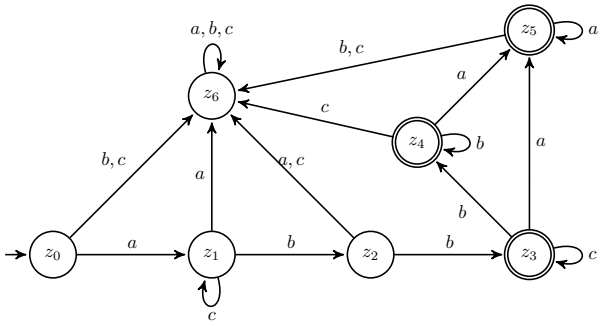
Theory of Computer Science

G. Röger
Spring Term 2019

University of Basel
Computer Science

Exercise Sheet 4 — Solutions

Exercise 4.1 (DFA and regular grammar; 2 + 2 marks)
Consider the following DFA M:



(a) Which language does the DFA accept?

Solution:

$\mathcal{L}(M) = \{ac^{n_1}bbc^{n_2}b^{n_3}a^{n_4} \mid n_1, n_2, n_3, n_4 \geq 0\}$

(b) Specify a *regular* grammar, which generates the same language.

Solution:

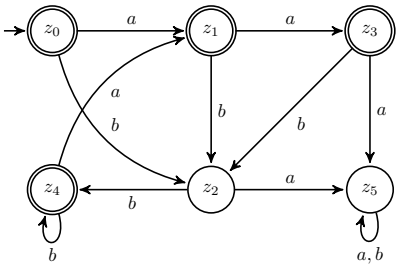
We construct a formal grammar as shown in the proof on slide 13 of slide set C02. Since no accept state can be reached from state z_6 we can directly omit all rules that contain the corresponding variable. Additionally we apply the transformation from slide 5/6 of slide set C02 in oder to remove the non-regular ε-rules. This results in a regular grammar $G = (\{a, b, c\}, \{A_0, \ldots, A_5\}, P, A_0)$ with the following rules in P:

$A_0 \to aA_1$	$A_1 \to cA_1$	$A_1 \to bA_2$	$A_2 \to bA_3$
$A_2 \to b$	$A_3 \to cA_3$	$A_3 \to c$	$A_3 \to aA_5$
$A_3 \to a$	$A_3 \to bA_4$	$A_3 \to b$	$A_4 \to bA_4$
$A_4 \to b$	$A_4 \to aA_5$	$A_4 \to a$	$A_5 \to aA_5$
$A_5 \to a$			

Exercise 4.2 (DFA; 2 Points)
Specify a deterministic finite automaton that accepts the language over $\Sigma = \{a, b\}$ where the words of the language have the following property:

When a occurs at the beginning of the word or after b, then there directly follows at most one further a. When b occurs at the beginning of the word or after a, then there directly follows at least one further b.

Solution:



Exercise 4.3 (Regular grammar and NFA; 1+1 Points)
Consider the language $L = \{w \in \{0,1\}^* \mid w \text{ ends with 01 or with 10}\}$.

(a) Specify a regular grammar that generates L.

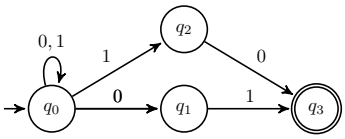
Solution:

$G = \langle\{0,1\}, \{S, A, B\}, P, S\rangle$, where P contains the following rules:

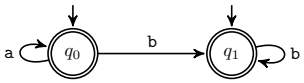
$$S \to 0S \mid 1S \mid 0A \mid 1B$$
$$A \to 1$$
$$B \to 0$$

(b) Draw the state diagram of an NFA with at most four states that accepts L.

Solution:



Exercise 4.4 (NFA; 0.5+1.5 Points)
Consider the following NFA M:



(a) What language $\mathcal{L}(M)$ does it accept?

Solution:

$\mathcal{L}(M) = \{\mathtt{a}^m\mathtt{b}^n \mid m, n \geq 0\}$

(b) Use the construction from the proof of the Theorem by Rabin & Scott to find a DFA that accepts the same language.

Solution: