Theory of Computer Science

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Exercise Sheet 12

Due: Wednesday, May 22, 2019

Exercise 12.1 (LOOP programs, 1 mark)

Which function does the following program compute?

LOOP x_1 DO
  $x_1 := x_1 + 1$
END;
LOOP x_1 DO
  $x_1 := x_1 + 1$
END;
$x_0 := x_1$

Exercise 12.2 (LOOP-computability, 0.5 marks)

Consider the following function g that computes a modified modulo operation:

$$g(x, y) = \begin{cases} x \bmod y, & \text{if } y > 0 \\ \text{undefined}, & \text{otherwise.} \end{cases}$$

Is g LOOP-computable?

Exercise 12.3 (Alternative Definition of LOOP programs, 2 marks)

Show that with the following definition of LOOP' programs, we can compute exactly the same functions as with the definition of LOOP programs from the lecture:

LOOP' programs are inductively defined as follows:

- $x_i := x_j$ is a LOOP' program for every $i, j \in \mathbb{N}_0$ (*assignment*)
- $x_i := x_i + 1$ is a LOOP' program for every $i \in \mathbb{N}_0$ (*incrementation*)
- $x_i := x_i - 1$ is a LOOP' program for every $i \in \mathbb{N}_0$ (*modified decrementation*)
- If P_1 and P_2 are LOOP' programs, then so is $P_1; P_2$ (*composition*)
- If P is a LOOP' program, then so is
  LOOP x_i *DO* P *END*
  for every $i \in \mathbb{N}_0$ (*LOOP loop*)

Exercise 12.4 (Syntactic Sugar, 1.5 + 1.5 + 1.5 marks)

Simulate the following syntactical constructs for LOOP-programs (with obvious semantics) by using already known constructs. In addition to the base constructs of LOOP programs you may use the additional constructs introduced in chapter F1.

(a) **IF** $x_i > c$ **THEN** P **ELSE** P' **END**

(b) **IF** $x_i = x_j$ **THEN** P **END**

(c) **FOR** $x_i = 1$ **TO** c **DO** P **END**

In the above constructs P and P' are arbitrary LOOP-programs and $i, j, c \in \mathbb{N}_0$ are arbitrary natural numbers.

Exercise 12.5 (2 marks)

Specify a LOOP program that computes the exponentiation $f(x, y) = x^y$. You may use all syntactic sugar introduced in the lecture.