Theory of Computer Science

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Spring Term 2019

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Exercise Sheet 11

Due: Wednesday, May 15, 2019

Exercise 11.1 (Polynomial Reductions, 2.5 + 0.5 marks)
Consider the decision problem 3Coloring:

- *Given:* undirected graph $G = \langle V, E\rangle$
- *Question:* Is there a total function $f : V \to \{r, g, b\}$ such that $f(v) \neq f(w)$ for all $\{v, w\} \in E$?

and the decision problem 3SAT:

- *Given:* a propositional formula φ in conjunctive normal form with *at most* 3 literals per clause
- *Question:* is φ satisfiable?

(a) Show that 3Coloring $\leq_p$ 3SAT.

(b) What can we say about 3Coloring, knowing that 3SAT is NP-complete?

Exercise 11.2 (NP-completeness, 2+2 marks)
Consider the decision problem HittingSet:

- *Given:* A finite set T, a set of sets $S = \{S_1, \dots, S_n\}$ with $S_i \subseteq T$ for all $i \in \{1, \dots, n\}$, a natural number $K \in \mathbb{N}_0$ with $K \leq |T|$.
- *Question:* Is there a set H with at most K elements that contains at least one element from each set in S?

(a) Prove that HittingSet is in NP by specifying a non-deterministic algorithm for HittingSet whose runtime is limited by a polynomial in $n|T|$.

(b) Prove that HittingSet is NP-complete. You may use without proof that the problem VertexCover (from chapter E5) is NP-complete.

Exercise 11.3 (NP-hardness, 3 marks)
Consider the following decision problems:

IndSet:

- *Given:* Undirected graph $G = \langle V, E\rangle$, number $k \in \mathbb{N}_0$
- *Question:* Does G contain an independent set of size k or larger,
i.e., is there a set $I \subseteq V$ with $|I| \geq k$ and $\{u, v\} \notin E$ for all $u, v \in I$?

SetPacking:

- *Given:* Finite set M, set $\mathcal{S} = \{S_1, \dots, S_n\}$ with $S_i \subseteq M$ for all $i \in \{1, \dots, n\}$, number $k \in \mathbb{N}_0$
- *Question:* Is there a set $\mathcal{S}' \subseteq \mathcal{S}$ with $|\mathcal{S}'| \geq k$, such that all sets in $\mathcal{S}'$ are pairwise disjoint, i.e., for all $S_i, S_j \in \mathcal{S}'$ with $S_i \neq S_j$ it holds that $S_i \cap S_j = \emptyset$?

Prove that SetPacking is NP-hard. You may use that the problem IndSet is NP-complete.