Theory of Computer Science

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Exercise Sheet 10

Due: Wednesday, May 8, 2019

Exercise 10.1 (1+1+1.5+1.5 marks)

Prove or refute the following statements. In all cases, specify a short proof (2–3 sentences are sufficient).

(a) Let X be an NP-hard problem and Y a problem with $X \leq_{\mathrm{p}} Y$. Then Y is NP-hard.

(b) Let X be an NP-hard problem. If there is a deterministic polynomial algorithm for X, then there also is a deterministic polynomial algorithm for DirHamiltonCycle.

(c) There are NP-complete problems X and Y where there is a deterministic polynomial algorithm for X but not for Y.

(d) Let $Y \subseteq \Sigma^*$ be any problem with $Y \neq \emptyset$ and $Y \neq \Sigma^*$. Then $X \leq_{\mathrm{p}} Y$ holds for all $X \in \mathrm{P}$.

Exercise 10.2 (Polynomial Reduction, 4 + 1 marks)

A *Hamilton path* is defined analogously to a Hamilton cycle (see chapter E1) with the only difference that we look for a simple path instead of a cycle. More formally: a Hamilton path in a directed graph $\langle V, E\rangle$ is a sequence of vertices $\pi = \langle v_1, \ldots, v_n\rangle$ that defines a path ($\langle v_i, v_{i+1}\rangle \in E$ for all $1 \leq i < n$) and contains every vertex in the graph exactly once.
Consider the decision problem DirHamiltonPath:

- *Given:* directed graph $G = \langle V, E\rangle$
- *Question:* Does G contain a Hamilton path?

(a) Prove that DirHamiltonPath is NP-hard. You can use without proof that DirHamiltonCycle is NP-complete.

(b) Is DirHamiltonPath NP-complete? Justify your answer.