

Theory of Computer Science

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Exercise Sheet 9

Due: Wednesday, May 1, 2019

Exercise 9.1 (Rice's Theorem, 0.5+0.5+0.5+0.5 marks)

For which of the following languages does Rice's theorem show that the language is undecidable? For each language where Rice's theorem can be used, specify the subset of Turing-computable functions \mathcal{S} for which you use the theorem.

Hint: You do not have to write down any proofs. If Rice's theorem is applicable, specify the set \mathcal{S} , otherwise give a short reason (1 sentence) why Rice's theorem is not applicable.

- (a) $L = \{w \in \{0, 1\}^* \mid M_w \text{ computes the binary multiplication function}\}$
- (b) $L = \{w \in \{0, 1\}^* \mid \text{The output of } M_w \text{ started on the empty tape contains } 0101\}$
- (c) $L = \{w \in \{0, 1\}^* \mid M_w \text{ stops for at least one input after more than 10 steps with a valid output}\}$
- (d) $L = \{w \in \{0, 1\}^* \mid M_w \text{ computes a binary function over the natural numbers}\}$

Exercise 9.2 (Non-deterministic algorithms, 2.5 marks)

Consider the decision problem VERTEXCOVER:

- *Given:* undirected graph $G = \langle V, E \rangle$, number $K \in \mathbb{N}_0$
- *Question:* Does G have a vertex cover of size at most K , i.e., a set of vertices $S \subseteq V$ with $|S| \leq K$ and $\{u, v\} \cap S \neq \emptyset$ for all $\{u, v\} \in E$?

Specify a non-deterministic algorithm for VERTEXCOVER, whose runtime is polynomial in the size of the input. Explain why the algorithm is correct and its runtime is polynomial.

Exercise 9.3 (Non-deterministic algorithms, 2.5+3 marks)

Consider the decision problem CLIQUE:

- *Given:* undirected graph $G = \langle V, E \rangle$, number $K \in \mathbb{N}_0$
 - *Question:* Does G contain a clique of size K or more, i.e., a set of nodes $C \subseteq V$ with $|C| \geq K$ and $\{u, v\} \in E$ for all $u, v \in C$ with $u \neq v$?
- (a) Specify a non-deterministic algorithm for CLIQUE, whose runtime is limited by a polynomial in $|V| + |E|$. Explain why the algorithm's runtime is polynomial.
 - (b) Specify a deterministic algorithm for CLIQUE and analyse its runtime in O -notation.

For exercises 2 and 3 you can use any common programming concepts in your answer. High-level pseudo code is sufficient as long as it can be easily seen that each step runs in polynomial time. Use the GUESS statements from the lecture for non-deterministic statements.