Theory of Computer Science

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Exercise Sheet 8

Due: Wednesday, April 24, 2019

Exercise 8.1 (Transitivity of Reductions, 1 point)

Show for any languages A, B and C: if $A \leq B$ and $B \leq C$, then $A \leq C$.

Exercise 8.2 (Undecidability of the emptiness problem, 4 points)

The *emptiness problem* EMPTINESS for general (type-0) grammars is defined as:

Given a general grammar G, is $\mathcal{L}(G) = \emptyset$?

Prove that EMPTINESS is undecidable.

Hints: you can use without proof that there is a computable function that transforms a given type-0 grammar G to a DTM M_G with $\mathcal{L}(M_G) = \mathcal{L}(G)$. Likewise, there is a computable function that transforms a given DTM M to a type-0 grammar G_M with $\mathcal{L}(M) = \mathcal{L}(G_M)$.
Use Rice's theorem in an appropriate way to show the undecidability.

Exercise 8.3 (Undecidability of intersection problem, 1 point)

The *intersection problem* INTERSECTION for general (type-0) grammars is defined as:

Given two general grammars G_1 and G_2, is $\mathcal{L}(G_1) \cap \mathcal{L}(G_2) = \emptyset$?

Show that INTERSECTION is undecidable using a reduction and the fact that EMPTINESS is undecidable.