Theory of Computer Science

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Exercise Sheet 7

Due: Monday, April 22, 2019

Exercise 7.1 (Composition of computable functions is computable; 2 points)

Let $f : \Sigma^* \to \Sigma^*$ and $g : \Sigma^* \to \Sigma^*$ be Turing-computable partial functions for an alphabet Σ. Show that the *composition* $(f \circ g) : \Sigma^* \to \Sigma^*$ is also Turing-computable.
In general the composition of two functions is defined as $(f \circ g)(x) = f(g(x))$. Specifically, the value $(f \circ g)(x)$ is undefined if $g(x)$ is undefined.

Exercise 7.2 (Enumerable Functions; 1.5+1.5 Points)

Let $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$. Specify total and computable functions $f : \mathbb{N}_0 \to \Sigma^*$ which recursively enumerate the following languages.

(a) $L_1 = \{\mathtt{a}^x\mathtt{b}^y \mid x, y \in \mathbb{N}_0\}$

(b) $L_2 = L_A \cup L_B$ where L_A and L_B are languages over Σ that are recursively enumerated by the functions f_A and f_B.

Exercise 7.3 (Decidability and Semi-Decidability; 0.5+0.5+1+1+1 Points)

Which of the following statements are true, which are false? In each case, specify a short proof (1 sentence) or a counter example. You can use all results from the lecture.

(a) Every decidable language is of type 0.

(b) If A is decidable then $\bar{A}$ is also decidable.

(c) Every language that is accepted by a Turing machine is decidable.

(d) Every language that can be described by a regular expression is decidable.

(e) Every decidable language is context-free.