Theory of Computer Science

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Exercise Sheet 3

Due: Wednesday, March 20, 2019

Exercise 3.1 (Refutation Theorem; 1.5 Points)
Prove the refutation theorem, that is, show for any set of formulas KB and any formula φ that

$$\mathrm{KB} \cup \{\varphi\} \text{ is unsatisfiable if and only if } \mathrm{KB} \models \neg\varphi.$$

Exercise 3.2 (Correctness of the Resolution Calculus; 1.5 Points)
Prove the correctness of the resolution rule

$$\frac{C_1 \cup \{L\},\ C_2 \cup \{\neg L\}}{C_1 \cup C_2},$$

by showing that for all interpretations $\mathcal{I}$ with $\mathcal{I} \models \bigvee_{\ell \in C_1 \cup \{L\}} \ell$ and $\mathcal{I} \models \bigvee_{\ell \in C_2 \cup \{\neg L\}} \ell$ it holds that $I \models \bigvee_{\ell \in C_1 \cup C_2} \ell$.

Exercise 3.3 (Resolution Calculus; 3 Points)
Consider the following knowledge base

$$\mathrm{KB} = \{(A \leftrightarrow \neg D), (\neg A \rightarrow (B \vee C)), ((A \rightarrow E) \wedge (B \vee C \vee F)), (E \rightarrow (F \rightarrow (B \vee C))),$$
$$(C \rightarrow G), (G \rightarrow \neg C)\}.$$

Use the resolution calculus to show that $\mathrm{KB} \models (B \wedge \neg C)$.

Exercise 3.4 (Predicate Logic; 3 Points)
Consider the following predicate logic formula φ with the signature $\langle \{x, y\}, \{\mathrm{c}\}, \{\mathrm{f}, \mathrm{g}\}, \{\mathrm{P}\} \rangle$.

$$\varphi = (\neg \mathrm{P}(\mathrm{c}) \wedge \forall x \exists y \, ((\mathrm{f}(y) = \mathrm{g}(x)) \wedge \mathrm{P}(y)))$$

Specify a model $\mathcal{I}$ of φ with $\mathcal{I} = \langle U, \cdot^{\mathcal{I}} \rangle$ and $\mathcal{U} = \{u_1, u_2, u_3\}$. Prove that $\mathcal{I} \models \varphi$. Why is no variable assignment α required to specify a model of φ?

Exercise 3.5 (Predicate logic; 1 Point)
Consider the formula φ over a signature with predicate symbols P (1-ary), Q (2-ary) and R (3-ary), the 1-ary function symbol f, the constant symbol c and the variable symbols x, y and z.

$$\varphi = (\forall x \exists y \, (\mathrm{P}(z) \rightarrow \mathrm{Q}(y, x)) \vee \neg \exists y \, \mathrm{R}(\mathrm{c}, x, \mathrm{f}(y)))$$

Mark all occurrences of free variables in φ. *Additionaly* specify the set of free variables of φ (without proof).