Theory of Computer Science

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Spring Term 2019

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Exercise Sheet 2

Due: Wednesday, March 6, 2019

Exercise 2.1 (Semantics; 0.5+0.5+1+1+1 Points)
Consider the propositional formula φ over $\{A, B, C, D, E, F\}$:

$$\varphi = ((F \lor ((\neg B \leftrightarrow ((C \land A) \rightarrow \neg B)) \lor (D \rightarrow E))) \rightarrow (A \rightarrow \neg F))$$

(a) How many lines would be needed for a truth table for φ?

(b) Formula φ is an implication. Specify the truth table for the general implication formula $\varphi \rightarrow \psi$. Attention: You should **not** specify the truth table of φ.

(c) Specify a model $\mathcal{I}$ for φ and prove without truth table that $\mathcal{I} \models \varphi$.

(d) Specify an assignment $\mathcal{I}$ with $\mathcal{I} \not\models \varphi$ and prove that $\mathcal{I}$ has the desired property without a truth table.

(e) Which of the properties *satisfiable*, *unsatisfiable*, *valid*, and *falsifiable* are true for φ? Justify your answer for each of the four properties.

Hint: The proofs for this exercises are fairly short (4 and 6 steps, respectively). If you need a considerably larger amount of steps, rethink your solution and try to find an easier proof. The solution of part (b) may help you identify the requirements for $\mathcal{I}$.

Exercise 2.2 (Equivalences; 1.5+1.5 Points)

(a) Transform the following formula into CNF by applying the equivalence rules shown in the lecture. For each step, only apply one equivalence rule and also specify it.

$$\varphi = ((A \rightarrow B) \leftrightarrow \neg C)$$

(b) Prove that the following formula is unsatisfiable by showing that $\varphi \equiv (A \land \neg A)$ holds. Use the equivalence rules from the lecture, only apply one rule for each step and specify the applied rule.

$$\varphi = \neg((A \land (\neg B \rightarrow A)) \lor \neg A)$$

Exercise 2.3 (Logical Consequence; 1.5+1.5 Points)
Consider the following formula set over $\{A, B, C\}$.

$$KB = \{(A \rightarrow \neg C), (A \lor \neg B), (\neg A \lor C)\}$$

(a) Does a model $\mathcal{I}$ of KB exist which is also a model for $\varphi = (A \lor B)$? Prove your statement.

(b) Prove that all models $\mathcal{I}$ of KB are also models of $\varphi = (\neg B \lor C)$.