

Theory of Computer Science

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Exercise meeting 11 — Solutions

Exercise 11.1 (WHILE Programs)

Which unary function does the following WHILE-program compute?

```
 $x_2 := 1;$   
 $x_3 := 0;$   
WHILE  $x_2 \neq 0$  DO  
  IF  $x_1 = x_3$  THEN  
     $x_2 := 0$   
  END;  
   $x_3 := x_3 + 2$   
END;  
 $x_0 := 1$ 
```

Solution:

The program computes the function

$$f(n) = \begin{cases} 1, & \text{if } n \text{ even} \\ \text{undefined}, & \text{if } n \text{ odd} \end{cases}$$

Exercise 11.2 (WHILE Programs)

(a) Specify a WHILE program that computes the following function:

$$f(x, y) = \begin{cases} \lfloor x/y \rfloor, & \text{if } y > 0 \\ \text{undefined}, & \text{otherwise} \end{cases}$$

You can use that $\lfloor a/b \rfloor = \lceil (a - (b - 1))/b \rceil$ for $b > 0$.

Solution:

```
 $x_3 := x_1 + 1;$   
 $x_3 := x_3 - x_2;$   
WHILE  $x_3 \neq 0$  DO  
   $x_3 := x_3 - x_2;$   
   $x_0 := x_0 + 1$   
END
```

(b) Specify a WHILE-program which computes the modulo operation

$$g(x, y) = \begin{cases} x \bmod y, & \text{if } y > 0 \\ \text{undefined}, & \text{otherwise.} \end{cases}$$

You may use the function f from exercise (a) and the multiplication \cdot in your solution.

Solution:

```
 $x_0 := f(x_1, x_2);$   
 $x_0 := x_0 \cdot x_2;$   
 $x_0 := x_1 - x_0$ 
```