

Theory of Computer Science

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Exercise meeting 11 — Solutions

Exercise 11.1 (WHILE Programs)

Which unary function does the following WHILE-program compute?

```
x2 := 1;  
x3 := 0;  
WHILE x2 ≠ 0 DO  
  IF x1 = x3 THEN  
    x2 := 0  
  END;  
  x3 := x3 + 2  
END;  
x0 := 1
```

Solution:

The program computes the function

$$f(n) = \begin{cases} 1, & \text{if } n \text{ even} \\ \text{undefined,} & \text{if } n \text{ odd} \end{cases}$$

Exercise 11.2 (WHILE Programs)

(a) Specify a WHILE program that computes the following function:

$$f(x, y) = \begin{cases} \lfloor x/y \rfloor, & \text{if } y > 0 \\ \text{undefined,} & \text{otherwise} \end{cases}$$

You can use that $\lfloor a/b \rfloor = \lceil (a - (b - 1))/b \rceil$ for $b > 0$.

Solution:

```
x3 := x1 + 1;  
x3 := x3 - x2;  
WHILE x3 ≠ 0 DO  
  x3 := x3 - x2;  
  x0 := x0 + 1  
END
```

(b) Specify a WHILE-program which computes the modulo operation

$$g(x, y) = \begin{cases} x \bmod y, & \text{if } y > 0 \\ \text{undefined,} & \text{otherwise.} \end{cases}$$

You may use the function f from exercise (a) and the multiplication \cdot in your solution.

Solution:

```
x0 := f(x1, x2);  
x0 := x0 · x2;  
x0 := x1 - x0
```