Theory of Computer Science

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Exercise meeting 5 — Solutions

Exercise 5.1

Specify a grammar G' in Chomsky normal form that generates the same language as the context-free grammar $G = \langle \Sigma, V, P, S \rangle$ with $\Sigma = \{a, b\}$, $V = \{S, X, Y, Z\}$ and the following rules in P:

$$S \to \varepsilon \quad S \to XZ \quad S \to Y \quad X \to Z \quad X \to aYa$$
$$Y \to bb \quad Y \to bY \quad Z \to X \quad Z \to bZ$$

Solution:

Step 1: eliminate rules of type $A \to B$ (with $A, B \in V$)
First, we eliminate the cycle: $X \to Z$ and $Z \to X$.

$$S \to \varepsilon \quad S \to \boldsymbol{RR} \quad S \to Y \quad \boldsymbol{R} \to aYa$$
$$Y \to bb \quad Y \to bY \quad \boldsymbol{R} \to b\boldsymbol{R}$$

Then we define a strict total order $<$ on the variables such that $A \to B \in P$ implies that $A < B$. The only such rule is $S \to Y$, so we can for example use $S < R < Y$. If we iterate from the largest to the smallest variable, we only need to do something when we reach S: replace $S \to Y$ with the corresponding direct rules.

$$S \to \varepsilon \quad S \to RR \quad \boldsymbol{S \to bb} \quad \boldsymbol{S \to bY}$$
$$R \to aYa \quad Y \to bb \quad Y \to bY \quad R \to bR$$

Step 2: eliminate rules with a terminal symbol which are not of type $A \to a$.

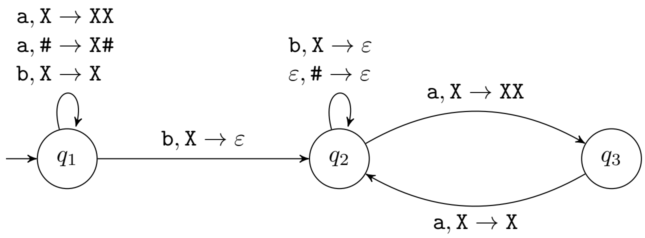
$$S \to \varepsilon \quad S \to RR \quad S \to \boldsymbol{BB} \quad S \to \boldsymbol{B}Y \quad R \to \boldsymbol{A}Y\boldsymbol{A}$$
$$Y \to \boldsymbol{BB} \quad Y \to \boldsymbol{B}Y \quad R \to \boldsymbol{B}R \quad \boldsymbol{A \to a} \quad \boldsymbol{B \to b}$$

Step 3: eliminate rules of type $A \to B_1 B_2 \dots B_k$ with $k > 2$. We only have one such rule: $R \to AYA$.
The final result is the grammar $G' = \langle \Sigma, V', P', S \rangle$ with terminal alphabet $\Sigma = \{a, b\}$, variables $V' = \{S, R, Y, A, B, K\}$ and the following rules in P':

$$S \to \varepsilon \quad S \to RR \quad S \to BB \quad S \to BY \quad R \to A\boldsymbol{K} \quad \boldsymbol{K} \to YA$$
$$Y \to BB \quad Y \to BY \quad R \to BR \quad A \to a \quad B \to b$$

Exercise 5.2

(a) Consider the PDA $M = \langle \{q_1, q_2, q_3\}, \{\mathtt{a}, \mathtt{b}\}, \{\mathtt{X}, \mathtt{\#}\}, \delta, q_1, \mathtt{\#}\rangle$ with the following transition function δ:



Prove that M accepts the word aababbaabb by specifying a sequence of configurations as defined in chapter C5.

Solution:

The following configurations prove that M accepts the word.

$$\langle q_1, \mathtt{aababbaabb}, \mathtt{\#}\rangle \vdash \langle q_1, \mathtt{ababbaabb}, \mathtt{X\#}\rangle \vdash \langle q_1, \mathtt{babbaabb}, \mathtt{XX\#}\rangle \vdash \langle q_1, \mathtt{abbaabb}, \mathtt{XX\#}\rangle$$
$$\vdash \langle q_1, \mathtt{bbaabb}, \mathtt{XXX\#}\rangle \vdash \langle q_2, \mathtt{baabb}, \mathtt{XX\#}\rangle \vdash \langle q_2, \mathtt{aabb}, \mathtt{X\#}\rangle \vdash \langle q_3, \mathtt{abb}, \mathtt{XX\#}\rangle \vdash \langle q_2, \mathtt{bb}, \mathtt{XX\#}\rangle$$
$$\vdash \langle q_2, \mathtt{b}, \mathtt{X\#}\rangle \vdash \langle q_2, \varepsilon, \mathtt{\#}\rangle \vdash \langle q_2, \varepsilon, \varepsilon\rangle$$

(b) Specify a PDA that accepts the language $L = \{(\mathtt{ab})^n\mathtt{c}\mathtt{a}^n\mathtt{c} \mid n \geq 0\}$ over $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$.

Solution:

The PDA $M = \langle \{q_1, q_2, q_3\}, \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}, \{\mathtt{N}, \mathtt{\#}\}, \delta, q_1, \mathtt{\#}\rangle$ accepts L if δ is defined as follows.