Theory of Computer Science

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Exercise meeting 4 — Solutions

Exercise 4.1

Consider the following regular expressions over the alphabet $\Sigma = \{0, 1\}$. For each regular expression, specify two words that are in the corresponding language and two words that are not in the corresponding language.

(a) $0|1^*|1\emptyset 0$

Solution:

The words 0 and 11 are in the language. Examples for words that are not in the language are 10 and `1101`.

(b) $1^*(\epsilon|0)(01)^*$

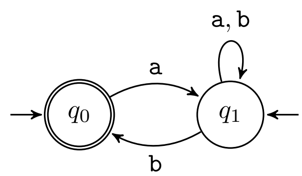
Solution:

The words 01 and 111001 are in the language. Examples for words that are not in the language are 100 and 11010.

Exercise 4.2

We consider regular languages over the alphabet $\Sigma = \{a, b\}$.

(a) Provide all reasons why the following finite automaton is an NFA but not a DFA.



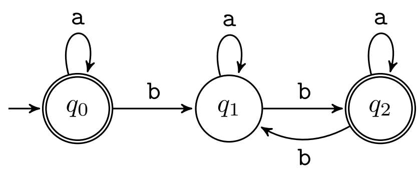
Solution:

- two start states
- no transition from q_0 with b
- two transitions from q_1 with b (to q_1 and q_0)

(b) Specify a deterministic finite automaton that accepts the language of all words over Σ with an even number of `b`s.

Solution:

This is one of several possible solutions:



Exercise 4.3

Are the following languages over $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}\}$ regular? If so, prove it by specifying a regular expression which describes the language. If not, prove it with help of the Pumping-Lemma.

(a) $L_1 = \{\mathtt{ab}^n\mathtt{c}^m\mathtt{d}^2 \mid n, m \in \mathbb{N}_0\}$

Solution:

L_1 is regular because it is described by the regular expression $\mathtt{ab}^*\mathtt{c}^*\mathtt{dd}$.

(b) $L_2 = \{w \in \{\mathtt{a}, \mathtt{b}\}^* \mid w \text{ contains as many } \mathtt{a}s \text{ as } \mathtt{b}s\}$

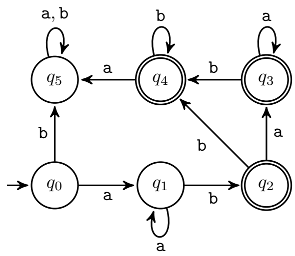
Solution:

Assume L_2 is regular. Let p be a pumping number of L_2. The word $w = \mathtt{a}^p\mathtt{b}^p$ is in L_2 and satisfies $|w| \geq p$. We know from the pumping lemma that there are words x, y and z with $w = xyz$, $|xy| \leq p$, $|y| \geq 1$ and $xy^iz \in L_2$ for all $i \geq 0$.

From $|xy| \leq p$ we know that xy can only consist of $\mathtt{a}$s. If we pump w smaller with $i = 0$, we get the word $w_0 := xy^0z = \mathtt{a}^{p-|y|}\mathtt{b}^p$. As $|y| \geq 1$ we know that $p - |y| < p$ and see that w_0 contains more $\mathtt{b}$ than $\mathtt{a}$s and hence $w_0 \notin L_2$. This is a contradiction to the pumping lemma and thus L_2 cannot be regular.

Exercise 4.4

Consider the following DFA M:



(a) Specify a regular expression that describes $\mathcal{L}(M)$.

Solution:

$\mathtt{aa}^*\mathtt{ba}^*\mathtt{b}^*$

(b) Specify the state diagram of an NFA with at most 4 states that accepts the same language.

Solution: