

Theory of Computer Science

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Exercise meeting 3 — Solutions

Exercise 3.1

Consider the grammar $G = \langle \Sigma, V, P, S \rangle$ with $\Sigma = \{a, b\}$, $V = \{S, A, B\}$ and the following rules in the set P :

$$\begin{aligned} S &\rightarrow ABB \\ S &\rightarrow \varepsilon \\ AB &\rightarrow AABBB \\ A &\rightarrow a \\ B &\rightarrow b \end{aligned}$$

Of what type(s) is G in the Chomsky hierarchy? What is $\mathcal{L}(G)$? Describe the language as simply as possible.

Solution:

This grammar is context-sensitive and of type 0. It is not context-free (due to rule $AB \rightarrow AABBB$) and therefore also not regular.

$$\mathcal{L}(G) = \{a^n b^{2n} \mid n \geq 0\}$$

Exercise 3.2

Specify a *complete description* of a formal grammar G that generates the language L that consists exactly of the string representations of all numbers from \mathbb{N}_0 in the decimal system. This means, L contains 0 and all non-empty words over $\{0, 1, \dots, 9\}$ that do not start with 0.

A formal grammar is a 4-tuple $G = \langle \Sigma, V, P, S \rangle$, remember to define all components of this tuple. Of what type(s) is your grammar in the Chomsky hierarchy?

Solution:

$G = \langle \{0, 1, \dots, 9\}, \{S, D, P\}, R, S \rangle$ with the following production rules R :

$$\begin{array}{lll} S \rightarrow 0 & S \rightarrow P & S \rightarrow PD \\ D \rightarrow DD & D \rightarrow P & D \rightarrow 0 \\ P \rightarrow 1 & P \rightarrow 2 & P \rightarrow 3 \\ P \rightarrow 4 & P \rightarrow 5 & P \rightarrow 6 \\ P \rightarrow 7 & P \rightarrow 8 & P \rightarrow 9 \end{array}$$

This grammar is context-free and therefore also context-sensitive and of type 0. It is not regular (e.g. due to rule $S \rightarrow PD$).