

# Theory of Computer Science

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Spring Term 2019

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## Exercise meeting 3 — Solutions

### Exercise 3.1

Consider the grammar  $G = \langle \Sigma, V, P, S \rangle$  with  $\Sigma = \{a, b\}$ ,  $V = \{S, A, B\}$  and the following rules in the set  $P$ :

$$\begin{aligned} S &\rightarrow ABB \\ S &\rightarrow \varepsilon \\ AB &\rightarrow AABBB \\ A &\rightarrow a \\ B &\rightarrow b \end{aligned}$$

Of what type(s) is  $G$  in the Chomsky hierarchy? What is  $\mathcal{L}(G)$ ? Describe the language as simply as possible.

#### Solution:

This grammar is context-sensitive and of type 0. It is not context-free (due to rule  $AB \rightarrow AABBB$ ) and therefore also not regular.

$$\mathcal{L}(G) = \{a^n b^{2^n} \mid n \geq 0\}$$

### Exercise 3.2

Specify a *complete description* of a formal grammar  $G$  that generates the language  $L$  that consists exactly of the string representations of all numbers from  $\mathbb{N}_0$  in the decimal system. This means,  $L$  contains 0 and all non-empty words over  $\{0, 1, \dots, 9\}$  that do not start with 0.

A formal grammar is a 4-tuple  $G = \langle \Sigma, V, P, S \rangle$ , remember to define all components of this tuple. Of what type(s) is your grammar in the Chomsky hierarchy?

#### Solution:

$G = \langle \{0, 1, \dots, 9\}, \{S, D, P\}, R, S \rangle$  with the following production rules  $R$ :

$$\begin{array}{lll} S \rightarrow 0 & S \rightarrow P & S \rightarrow PD \\ D \rightarrow DD & D \rightarrow P & D \rightarrow 0 \\ P \rightarrow 1 & P \rightarrow 2 & P \rightarrow 3 \\ P \rightarrow 4 & P \rightarrow 5 & P \rightarrow 6 \\ P \rightarrow 7 & P \rightarrow 8 & P \rightarrow 9 \end{array}$$

This grammar is context-free and therefore also context-sensitive and of type 0. It is not regular (e.g. due to rule  $S \rightarrow PD$ ).