

Theory of Computer Science

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Spring Term 2019

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Exercise meeting 8

Exercise 8.1

This exercise was a question in the exam in 2017.

Consider the following decision problems:

DIRHAMILTONPATH:

- *Given:* directed graph $G = \langle V, E \rangle$
- *Question:* Does G contain a Hamilton path?

DIRHAMILTONPATHWITHSTARTPOINT:

- *Given:* directed graph $G = \langle V, E \rangle$, start vertex $v_s \in V$
- *Question:* Does G contain a Hamilton path with start vertex v_s , i.e., a Hamilton path $\pi = \langle v_1, \dots, v_n \rangle$ with $v_1 = v_s$?

(a) Show $\text{DIRHAMILTONPATHWITHSTARTPOINT} \in \text{NP}$ by specifying a non-deterministic polynomial algorithm.

(b) Prove that $\text{DIRHAMILTONPATHWITHSTARTPOINT}$ is NP-hard. You may use that DIRHAMILTONPATH is NP-complete.

Reminder: A *Hamilton path* in a graph $\langle V, E \rangle$ is a vertex sequence $\pi = \langle v_1, \dots, v_n \rangle$ that defines a path $(\langle v_i, v_{i+1} \rangle \in E \text{ for all } 1 \leq i < n)$ and includes every graph vertex exactly once.