Foundations of Artificial Intelligence

42. Board Games: Alpha-Beta Search

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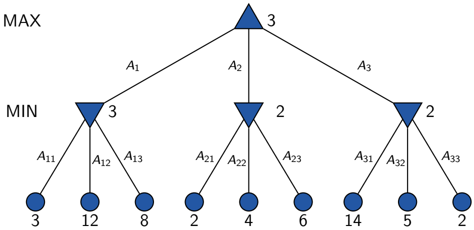
May 20, 2019

Board Games: Overview

chapter overview:

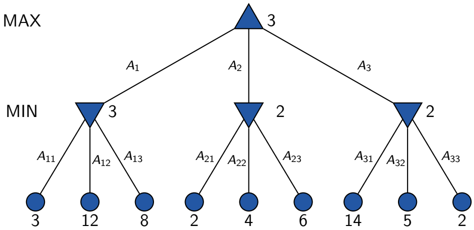
Alpha-Beta Search

Alpha-Beta Search



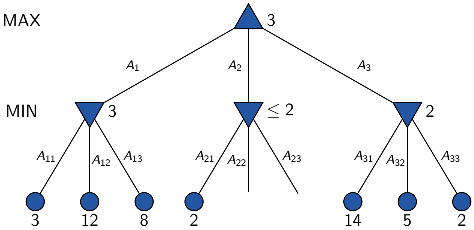
Can we save search effort?

Alpha-Beta Search

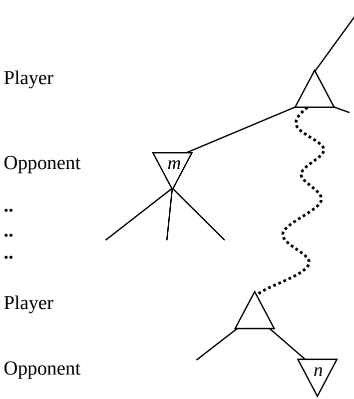


Can we save search effort?

We do not need to consider all the nodes!



Alpha-Beta Search: Generally



If $m > n$, then node with utility n will never be reached when playing perfectly!

Alpha-Beta Search: Idea

idea: Use two values α and β during minimax depth-first search, such that the following holds for every recursive call:

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- If the utility value in the current subtree is $\geq \beta$, then the subtree is not interesting because MIN will never enter it when playing perfectly.

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If $\alpha \geq \beta$ in the subtree, then the subtree is not interesting and does not have to be searched further (α-β pruning).

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Starting with $\alpha = -\infty$ and $\beta = +\infty$, alpha-beta search produces the identical result as minimax, with lower seach effort.

Alpha-Beta Search: Pseudo Code

- algorithm skeleton the same as minimax
- function signature extended by two variables α and β

function alpha-beta-main(p)

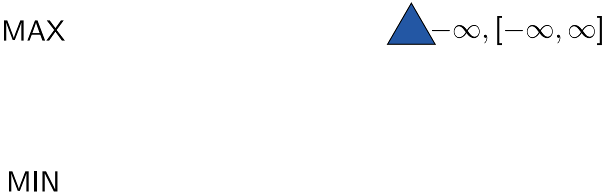
$\langle v, move\rangle :=$ alpha-beta$(p, -\infty, +\infty)$
return *move*

Alpha-Beta Search: Pseudo-Code

function alpha-beta(p, α, β)

```
if p is terminal position:
    return ⟨u(p), none⟩
initialize v and best_move                                  [as in minimax]
for each ⟨move, p′⟩ ∈ succ(p):
    ⟨v′, best_move′⟩ := alpha-beta(p′, α, β)
    update v and best_move                                  [as in minimax]
    if player(p) = MAX:
        if v ≥ β:
            return ⟨v, none⟩
        α := max{α, v}
    if player(p) = MIN:
        if v ≤ α:
            return ⟨v, none⟩
        β := min{β, v}
return ⟨v, best_move⟩
```

Alpha-Beta Search: Example



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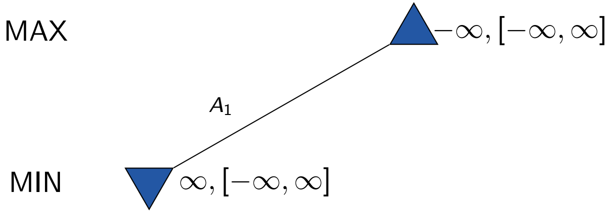

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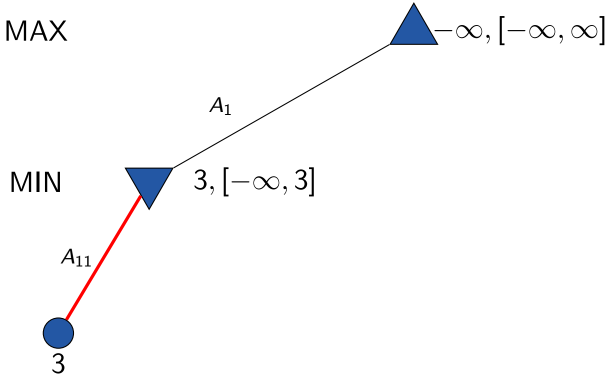

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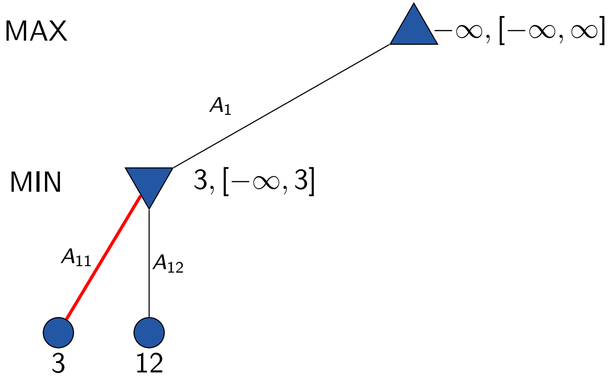

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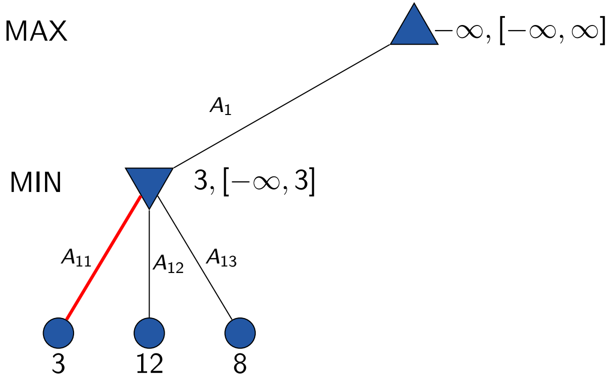

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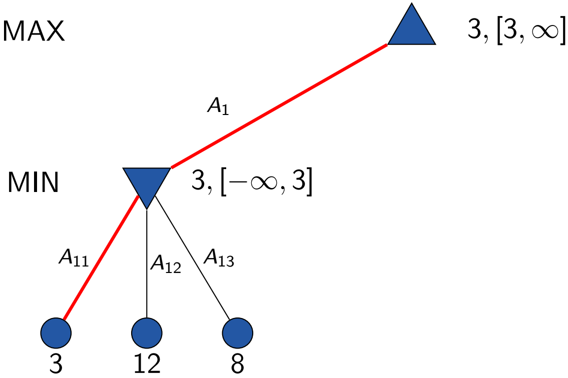

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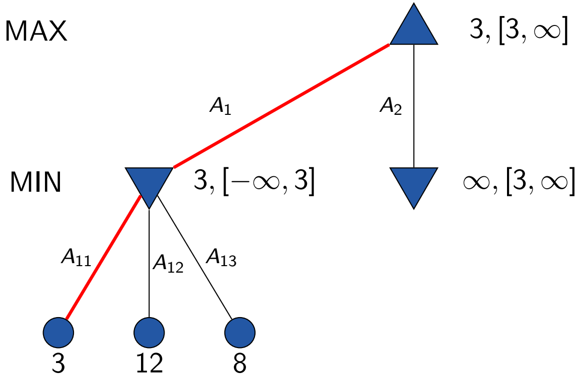

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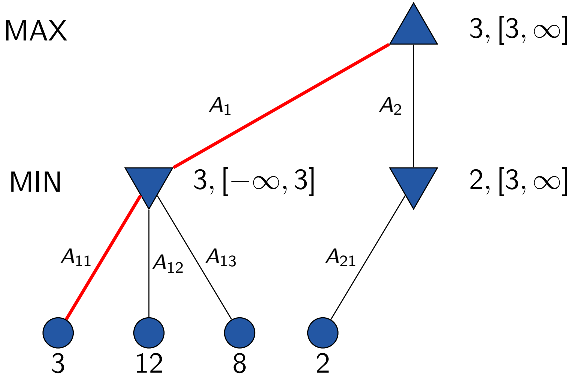

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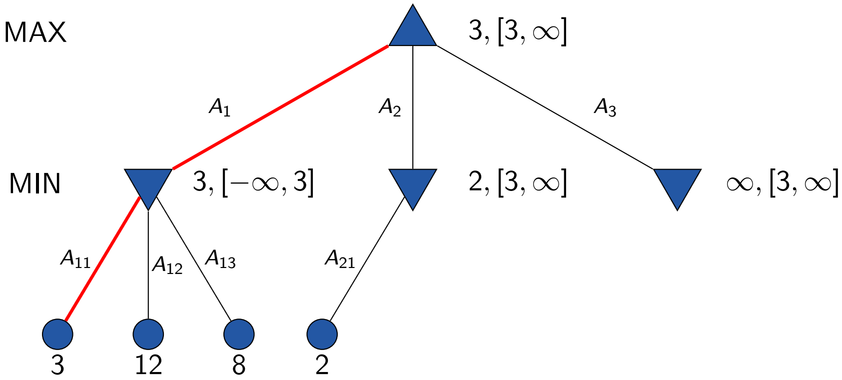

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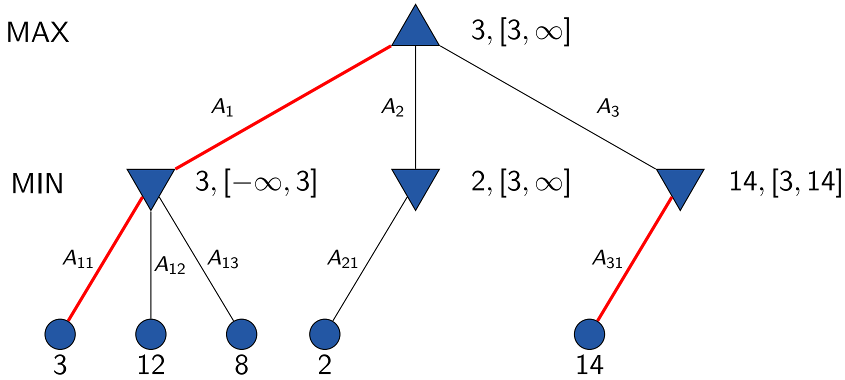

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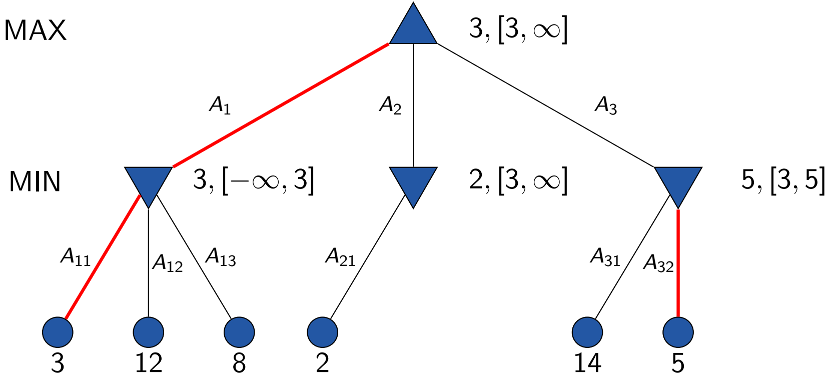

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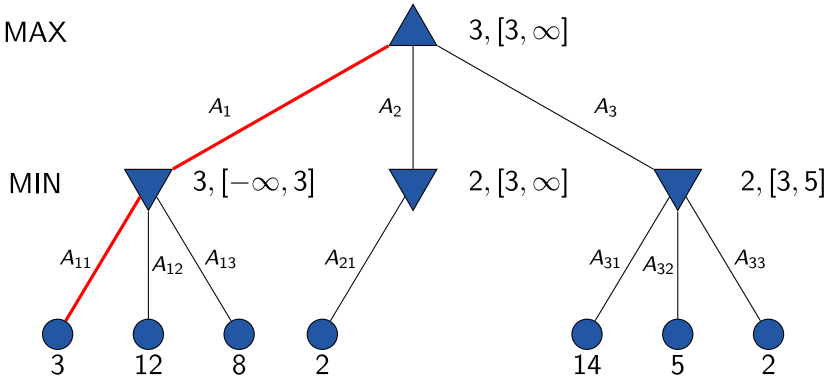
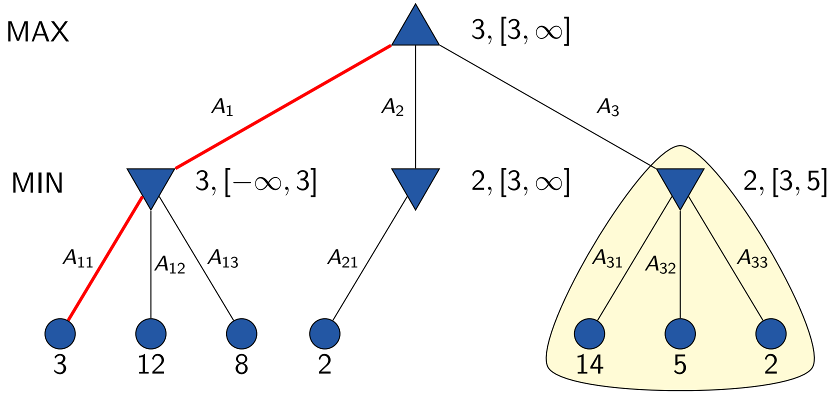


Move Ordering

Alpha-Beta Search: Example



If the last successor had been first, the rest of the subtree would have been pruned.

Move Ordering

idea: consider first the successors that are likely to be best.

- Domain-specific ordering function
  e.g. chess: captures $<$ threats $<$ forward moves $<$ backward moves
- Dynamic move-ordering
  - try first moves that have been good in the past
  - e.g. in iterative deepening search:
    best moves from previous iteration

How Much Do We Gain with Alpha-Beta Search?

assumption: uniform game tree, depth d, branching factor $b \geq 2$; MAX and MIN positions alternating

- perfect move ordering
  - best move at every position is considered first (this cannot be done in practice – Why?)
  - maximizing move for MAX, minimizing move for MIN
  - effort reduced from $O(b^d)$ (minimax) to $O(b^{d/2})$
  - doubles the search depth that can be achieved in same time
- random move ordering
  - effort still reduced to $O(b^{3d/4})$ (for moderate b)

In practice, it is often possible to get close to the optimum.

Summary

Summary

alpha-beta search

- stores which utility both players can force somewhere else in the game tree
- exploits this information to avoid unnecessary computations
- can have significantly lower search effort than minimax
- best case: search twice as deep in the same time