

# Foundations of Artificial Intelligence

## 39. Automated Planning: Landmark Heuristics

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## May 8, 2019 — 39. Automated Planning: Landmark Heuristics

### 39.1 Finding Landmarks

### 39.2 The LM-Cut Heuristic

### 39.3 Summary

## Automated Planning: Overview

### Chapter overview: automated planning

- ▶ 33. Introduction
- ▶ 34. Planning Formalisms
- ▶ 35.–36. Planning Heuristics: Delete Relaxation
- ▶ 37 Planning Heuristics: Abstraction
- ▶ 38.–39. Planning Heuristics: Landmarks
  - ▶ 38. Landmarks
  - ▶ 39. Landmark Heuristics

## Formalism and Example

- ▶ As in the previous chapter, we consider delete-free planning tasks in normal form.
- ▶ We continue with the example from the previous chapter:

### Example

actions:

- ▶  $a_1 = i \xrightarrow{3} x, y$
- ▶  $a_2 = i \xrightarrow{4} x, z$
- ▶  $a_3 = i \xrightarrow{5} y, z$
- ▶  $a_4 = x, y, z \xrightarrow{0} g$

landmark examples:

- ▶  $A = \{a_4\}$  (cost = 0)
- ▶  $B = \{a_1, a_2\}$  (cost = 3)
- ▶  $C = \{a_1, a_3\}$  (cost = 3)
- ▶  $D = \{a_2, a_3\}$  (cost = 4)

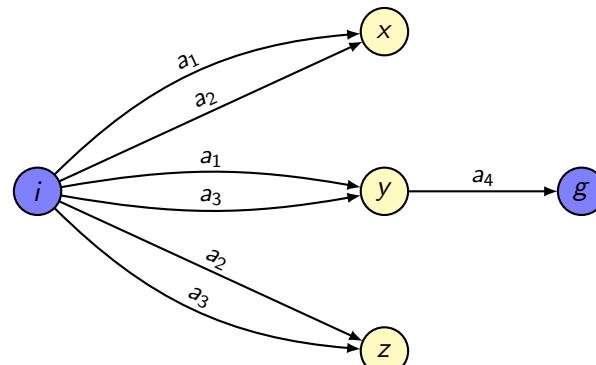
## 39.1 Finding Landmarks

### Example: Justification Graph

#### Example

pcf  $P$ :  $P(a_1) = P(a_2) = P(a_3) = i, P(a_4) = y$

$$\begin{aligned} a_1 &= i \xrightarrow{3} x, y \\ a_2 &= i \xrightarrow{4} x, z \\ a_3 &= i \xrightarrow{5} y, z \\ a_4 &= x, y, z \xrightarrow{0} g \end{aligned}$$



## Justification Graphs

#### Definition (precondition choice function)

A **precondition choice function** (pcf)  $P : A \rightarrow V$  maps every action to one of its preconditions.

#### Definition (justification graph)

The **justification graph** for pcf  $P$  is a directed graph with annotated edges.

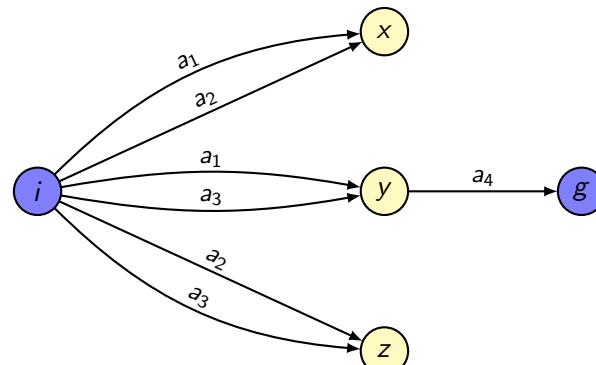
- ▶ **vertices**: the variables  $V$
- ▶ **edges**:  $P(a) \xrightarrow{a} e$  for every action  $a$ , every effect  $e \in add(a)$

### Example: Justification Graph

#### Example

pcf  $P$ :  $P(a_1) = P(a_2) = P(a_3) = i, P(a_4) = y$

$$\begin{aligned} a_1 &= i \xrightarrow{3} x, y \\ a_2 &= i \xrightarrow{4} x, z \\ a_3 &= i \xrightarrow{5} y, z \\ a_4 &= x, y, z \xrightarrow{0} g \end{aligned}$$



## Cuts

#### Definition (cut)

A **cut** in a justification graph is a subset  $C$  of its edges such that all paths from  $i$  to  $g$  contain an edge in  $C$ .

#### Proposition (cuts are landmarks)

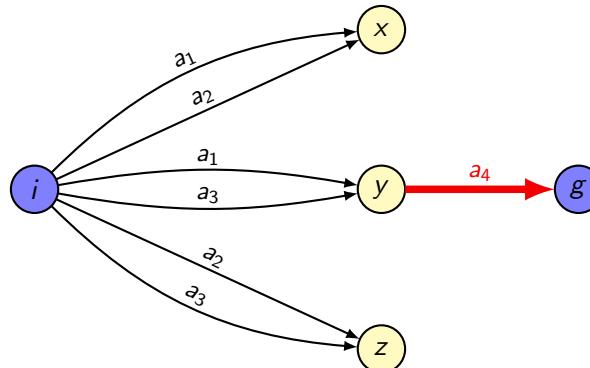
Let  $C$  be a cut in a justification graph for an arbitrary pcf. Then the edge annotations for  $C$  form a landmark.

## Example: Cuts in Justification Graphs

### Example

landmark  $A = \{a_4\}$  (cost = 0)

$$\begin{aligned} a_1 &= i \xrightarrow{3} x, y \\ a_2 &= i \xrightarrow{4} x, z \\ a_3 &= i \xrightarrow{5} y, z \\ a_4 &= x, y, z \xrightarrow{0} g \end{aligned}$$

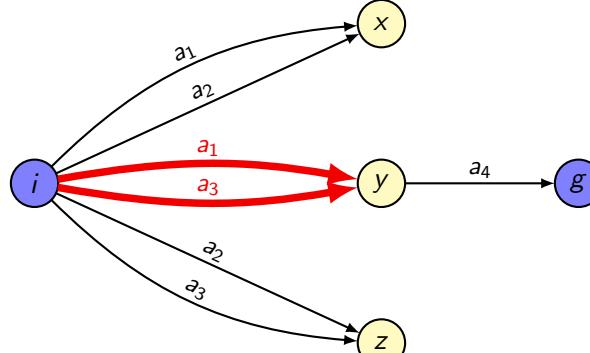


## Example: Cuts in Justification Graphs

### Example

landmark  $B = \{a_1, a_2\}$  (cost = 3)

$$\begin{aligned} a_1 &= i \xrightarrow{3} x, y \\ a_2 &= i \xrightarrow{4} x, z \\ a_3 &= i \xrightarrow{5} y, z \\ a_4 &= x, y, z \xrightarrow{0} g \end{aligned}$$

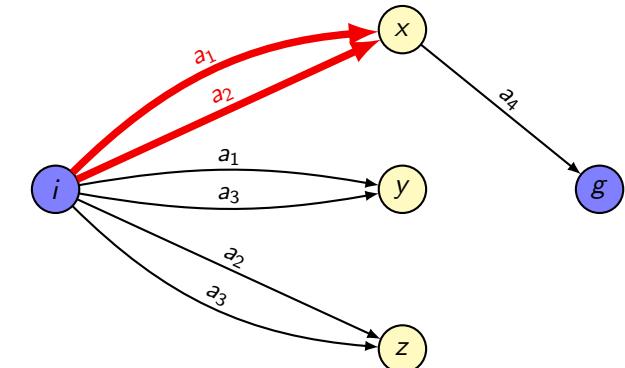


## Example: Cuts in Justification Graphs

### Example

landmark  $C = \{a_1, a_3\}$  (cost = 3)

$$\begin{aligned} a_1 &= i \xrightarrow{3} x, y \\ a_2 &= i \xrightarrow{4} x, z \\ a_3 &= i \xrightarrow{5} y, z \\ a_4 &= x, y, z \xrightarrow{0} g \end{aligned}$$

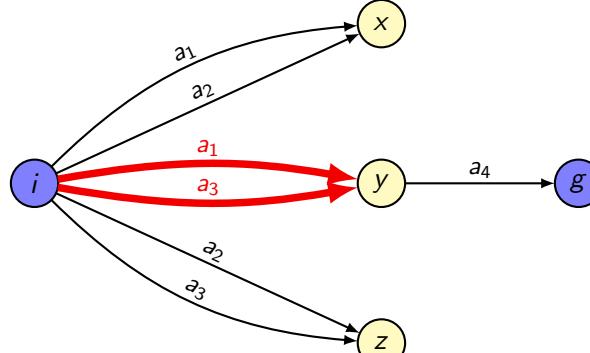


## Example: Cuts in Justification Graphs

### Example

landmark  $D = \{a_2, a_3\}$  (cost = 4)

$$\begin{aligned} a_1 &= i \xrightarrow{3} x, y \\ a_2 &= i \xrightarrow{4} x, z \\ a_3 &= i \xrightarrow{5} y, z \\ a_4 &= x, y, z \xrightarrow{0} g \end{aligned}$$

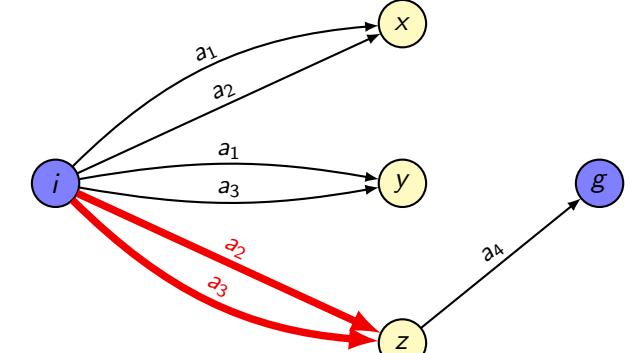


## Example: Cuts in Justification Graphs

### Example

landmark  $D = \{a_2, a_3\}$  (cost = 4)

$$\begin{aligned} a_1 &= i \xrightarrow{3} x, y \\ a_2 &= i \xrightarrow{4} x, z \\ a_3 &= i \xrightarrow{5} y, z \\ a_4 &= x, y, z \xrightarrow{0} g \end{aligned}$$



## Power of Cuts in Justification Graphs

- ▶ Which landmarks can be computed with the cut method?
- ▶ all interesting ones!

### Proposition (perfect hitting set heuristics)

Let  $\mathcal{L}$  be the set of all “cut landmarks” of a given planning task.  
Then  $h^{\text{MHS}}(I) = h^+(I)$  for  $\mathcal{L}$ .

↝ hitting set heuristic for  $\mathcal{L}$  is perfect.

proof idea:

- ▶ Show 1:1 correspondence of hitting sets  $H$  for  $\mathcal{L}$  and plans, i.e., each hitting set for  $\mathcal{L}$  corresponds to a plan, and vice versa.

## LM-Cut Heuristic: Motivation

- ▶ In general, there are exponentially many pcfs, hence computing all relevant landmarks is not tractable.
- ▶ The **LM-cut heuristic** is a method that chooses pcfs and computes cuts in a **goal-oriented** way.
- ▶ A cost partitioning is computed as a side effect and is usually not optimal.
- ▶ However, the cost partitioning can be computed efficiently and is optimal for planning tasks with uniform costs (i.e.,  $\text{cost}(a) = 1$  for all actions).
- ↝ currently one of the best admissible planning heuristics

## 39.2 The LM-Cut Heuristic

## The LM-Cut Heuristic

### $h^{\text{LM-cut}}$ : Helmert & Domshlak (2009)

Initialize  $h^{\text{LM-cut}}(I) := 0$ . Then iterate:

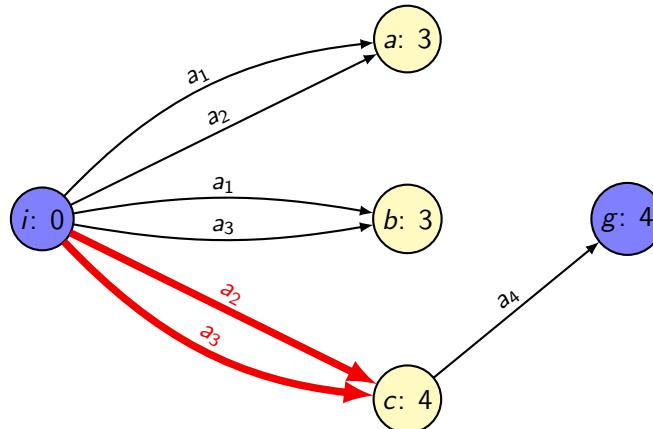
- ① Compute  $h^{\text{max}}$  values of the variables. Stop if  $h^{\text{max}}(g) = 0$ .
- ② Let  $P$  be a pcf that chooses preconditions with maximal  $h^{\text{max}}$  value. (Requires a tie-breaking policy.)
- ③ Compute the justification graph for  $P$ .
- ④ Compute a cut which guarantees  $\text{cost}(L) > 0$  for the corresponding landmark  $L$ . (We omit the details of how this is done.)
- ⑤ Increase  $h^{\text{LM-cut}}(I)$  by  $\text{cost}(L)$ .
- ⑥ Decrease  $\text{cost}(a)$  by  $\text{cost}(L)$  for all  $a \in L$ .

## Example: Computation of LM-Cut

### Example

round 1:  $P(a_4) = c \rightsquigarrow L = \{a_2, a_3\} [4]$

$a_1 = i \xrightarrow{3} a, b$   
 $a_2 = i \xrightarrow{4} a, c$   
 $a_3 = i \xrightarrow{5} b, c$   
 $a_4 = a, b, c \xrightarrow{0} g$

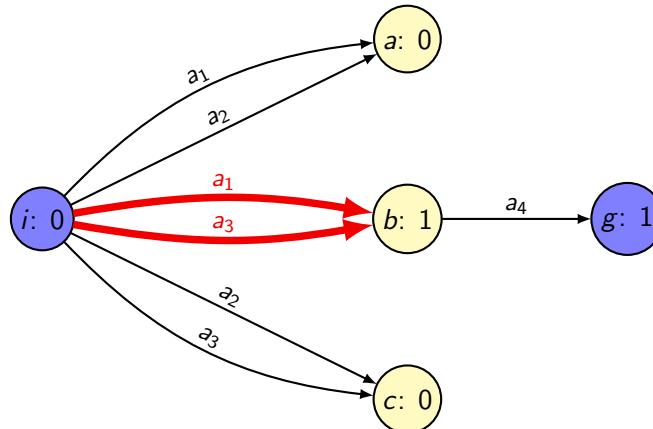


## Example: Computation of LM-Cut

### Example

round 2:  $P(a_4) = b \rightsquigarrow L = \{a_1, a_3\} [1]$

$a_1 = i \xrightarrow{3} a, b$   
 $a_2 = i \xrightarrow{0} a, c$   
 $a_3 = i \xrightarrow{1} b, c$   
 $a_4 = a, b, c \xrightarrow{0} g$

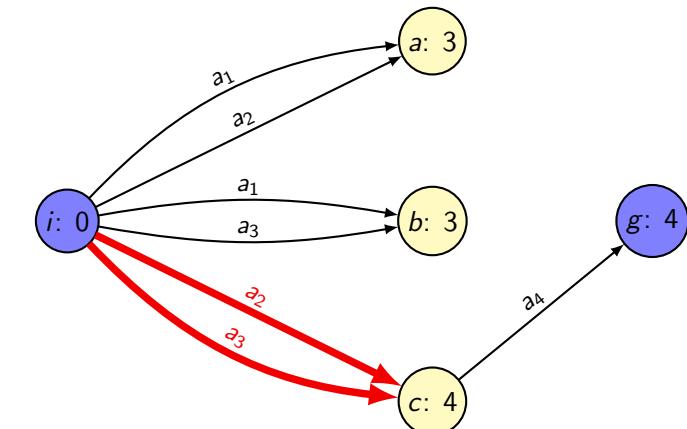


## Example: Computation of LM-Cut

### Example

round 1:  $P(a_4) = c \rightsquigarrow L = \{a_2, a_3\} [4] \rightsquigarrow h^{\text{LM-cut}}(I) := 4$

$a_1 = i \xrightarrow{3} a, b$   
 $a_2 = i \xrightarrow{0} a, c$   
 $a_3 = i \xrightarrow{1} b, c$   
 $a_4 = a, b, c \xrightarrow{0} g$



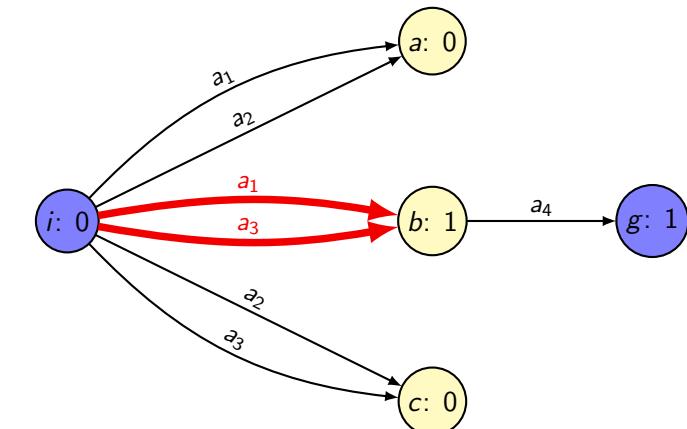
## Example: Computation of LM-Cut

## Example: Computation of LM-Cut

### Example

round 2:  $P(a_4) = b \rightsquigarrow L = \{a_1, a_3\} [1] \rightsquigarrow h^{\text{LM-cut}}(I) := 4 + 1 = 5$

$a_1 = i \xrightarrow{2} a, b$   
 $a_2 = i \xrightarrow{0} a, c$   
 $a_3 = i \xrightarrow{0} b, c$   
 $a_4 = a, b, c \xrightarrow{0} g$

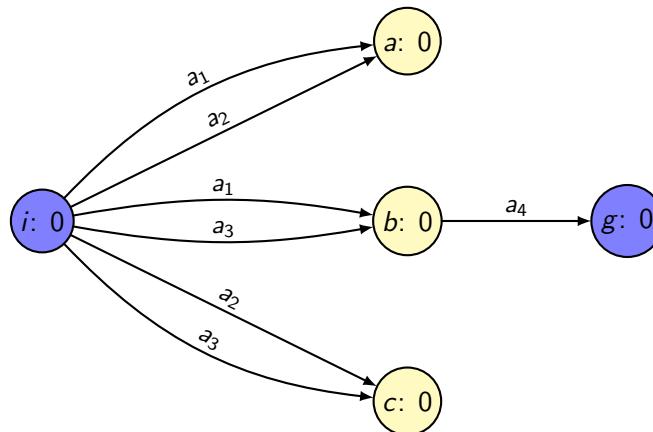


## Example: Computation of LM-Cut

### Example

round 3:  $h^{\max}(g) = 0 \rightsquigarrow \text{done!} \rightsquigarrow h^{\text{LM-cut}}(I) = 5$

$$\begin{aligned} a_1 &= i \xrightarrow{2} a, b \\ a_2 &= i \xrightarrow{0} a, c \\ a_3 &= i \xrightarrow{0} b, c \\ a_4 &= a, b, c \xrightarrow{0} g \end{aligned}$$



## 39.3 Summary

### Summary

- ▶ **Cuts in justification graphs** are a general method to find landmarks.
- ▶ Hitting sets over **all cut landmarks** yield a **perfect heuristic** for delete-free planning tasks.
- ▶ The **LM-cut heuristic** is an admissible heuristic based on these ideas.