

# Foundations of Artificial Intelligence

## 13. State-Space Search: Heuristics

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# State-Space Search: Overview

## Chapter overview: state-space search

- 5.–7. Foundations
- 8.–12. Basic Algorithms
- 13.–19. Heuristic Algorithms
  - 13. Heuristics
  - 14. Analysis of Heuristics
  - 15. Best-first Graph Search
  - 16. Greedy Best-first Search,  $A^*$ , Weighted  $A^*$
  - 17. IDA<sup>\*</sup>
  - 18. Properties of  $A^*$ , Part I
  - 19. Properties of  $A^*$ , Part II

# Introduction

# Informed Search Algorithms

- search algorithms considered so far: **blind**  
because they do not use any aspects of the problem to solve other than its formal definition (state space)
  - **problem**: scalability
    - ↪ prohibitive time and space requirements already for seemingly **simple** problems
  - **idea**: try to find (problem-specific) criteria to distinguish **good** and **bad states**
    - ↪ **prefer good states**
- ↪ **informed** (“heuristic”) search algorithms

# Heuristics

# Heuristics

## Definition (heuristic)

Let  $\mathcal{S}$  be a state space with states  $S$ .

A **heuristic function** or **heuristic** for  $\mathcal{S}$  is a function

$$h : \mathcal{S} \rightarrow \mathbb{R}_0^+ \cup \{\infty\},$$

mapping each state to a non-negative number (or  $\infty$ ).

# Heuristics: Intuition

**idea:**  $h(s)$  estimates distance (= cost of cheapest path) from  $s$  to closest goal state

- heuristics can be **arbitrary** functions
- **intuition:** the closer  $h$  is to true goal distance, the more efficient the search using  $h$

Heuristics are sometimes defined for **search nodes** instead of states, but this increased generality is rarely useful. (**Why?**)

# Why “Heuristic”?

## What does “heuristic” mean?

- **heuristic**: from ancient Greek  $\acute{\epsilon}\upsilon\text{ρισκω}$  (= I find)  
     $\rightsquigarrow$  **compare**:  $\acute{\epsilon}\upsilon\text{ρηκα!}$
- popularized by George Pólya: How to Solve It (1945)
- in computer science often used for:  
    rule of thumb, inexact algorithm
- in state-space search **technical term** for **goal distance estimator**



# Representation of Heuristics

In our black box model, heuristics are an additional element of the state space interface:

## State Spaces as Black Boxes (Extended)

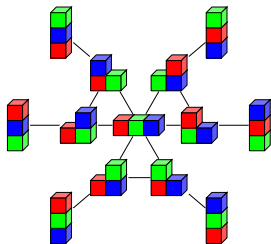
- `init()`
- `is_goal(s)`
- `succ(s)`
- `cost(a)`
- **`h(s)`**: heuristic value for state `s`  
  **result**: non-negative integer or  $\infty$

# Examples

# Example: Blocks World

possible heuristic:

count blocks  $x$  that currently lie on  $y$   
and must lie on  $z \neq y$  in the goal  
(including case where  $y$  or  $z$  is the table)

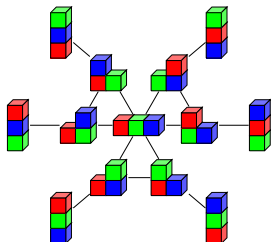


# Example: Blocks World

possible heuristic:

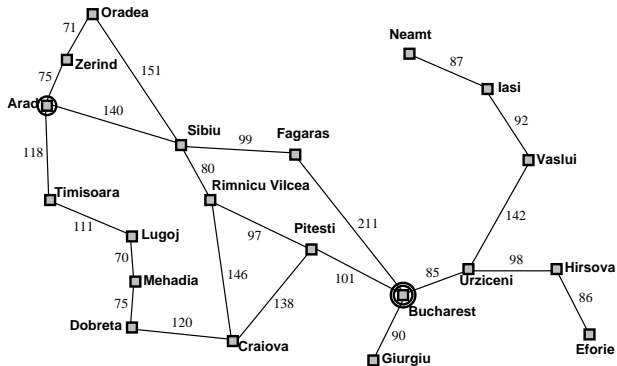
count blocks  $x$  that currently lie on  $y$   
and must lie on  $z \neq y$  in the goal  
(including case where  $y$  or  $z$  is the table)

How accurate is this heuristic?



# Example: Route Planning in Romania

possible heuristic: straight-line distance to Bucharest



Arad	366
Bucharest	0
Craiova	160
Drobeta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	100
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

# Example: Missionaries and Cannibals

## Setting: Missionaries and Cannibals

- Six people must cross a river.
- Their rowing boat can carry one or two people across the river at a time (it is too small for three).
- Three people are missionaries, three are cannibals.
- Missionaries may never stay with a majority of cannibals.

possible heuristic: number of people on the wrong river bank

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↪ with our formulation of states as triples  $\langle m, c, b \rangle$ :  
 $h(\langle m, c, b \rangle) = m + c$

# Summary



# Summary

- **heuristics** estimate distance of a state to the goal
  - can be used to **focus** search on **promising** states
- ~> **soon**: search algorithms that use heuristics