Foundations of Artificial Intelligence

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Spring Term 2019

Exercise Sheet 9
Due: May 1, 2019

Exercise 9.1 (1 marks)

Consider the propositional formula

$$\varphi = ((A \vee B) \to C).$$

Check for each of the following properties of propositional formulas if φ has that property. Justify your answers. Where possible, provide a suitable interpretation.

(a) satisfiable

(b) falsifiable

(c) valid

Exercise 9.2 (2+2 marks)

Compile the formulas below to CNF by applying the logical equivalences from slide 24 of chapter 29 in the print version of the lecture. You may additionally consider commutativity (1) and associativity (2):

$$\varphi \wedge \psi \equiv \psi \wedge \varphi \text{ and } \varphi \vee \psi \equiv \psi \vee \varphi \tag{1}$$
$$((\varphi \wedge \psi) \wedge \eta) \equiv (\varphi \wedge (\psi \wedge \eta)) \text{ and } ((\varphi \vee \psi) \vee \eta) \equiv (\varphi \vee (\psi \vee \eta)). \tag{2}$$

Provide all intermediate formulas that result from applying an equivalence transformation. If you use commutativity or associativity, you do not need to provide the corresponding steps.

(a) $(\neg P \vee Q) \to R$

(b) $(\neg(A \to B)) \vee (\neg C \wedge A)$

Exercise 9.3 (2 marks)

In the lecture, we have shown that the resolution method can be used for reasoning by a reduction to testing unsatisfiability. In this way, use the resolution method to show that $C \wedge \neg D$ follows logically from $\{\{A, B, C\}, \{\neg A, \neg B, D\}, \{A, \neg B, C\}, \{B, C, D\}, \{\neg D, F\}, \{E, \neg F\}, \{\neg D, \neg E\}\}$. Compare the number of required resolution steps with the number of entries that would have been necessary to show the same result with a truth table.

Exercise 9.4 (2.5+2.5 marks)

Use DPLL to show that the following sets of clauses are satisfiable or unsatisfiable. For the variable selection strategy, always choose a variable that occurs in the highest number of clauses (use the alphabetical ordering to break ties). If you use the splitting rule for some variable v, always consider the assignment $v \mapsto \mathbf{T}$ first. Provide the same intermediate results and applied rules of DPLL that were also given in the example in the lecture.

(a) $\{\{P, \neg Q\}, \{\neg P, Q\}, \{Q, \neg R\}, \{S\}, \{\neg S, \neg Q, \neg R\}, \{S, R\}\}$

(b) $\{\{P, Q, S, T\}, \{P, S, \neg T\}, \{Q, \neg S, T\}, \{P, \neg S, \neg T\}, \{P, \neg Q\}, \{\neg R, \neg P\}, \{R\}\}$

Important: Solutions should be submitted in groups of two students. However, only one student should upload the solution. Please provide both student names on each file and each page you submit. We can only accept a single PDF or a ZIP file containing *.java or *.pddl files and a single PDF.