Foundations of Artificial Intelligence

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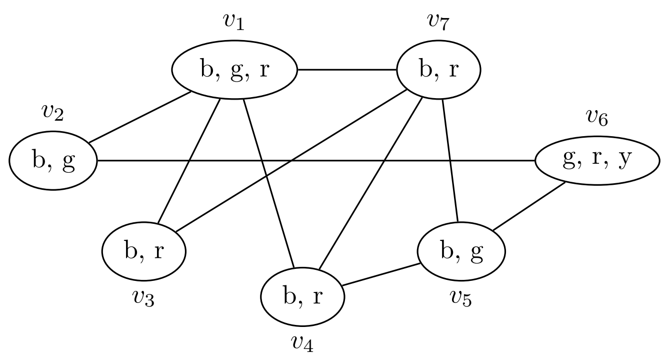
Exercise Sheet 7

Due: April 17, 2019

Exercise 7.1 (1+2 marks)

Consider the constraint network for the graph coloring problem that has been introduced on slide 10 of chapter 24 in the print version of the lecture slides:



Use the following static strategies on variable and value orderings:

- Variable ordering:
  - select a variable according to the *minimum remaining values* variable ordering criterion;
  - if there is more than one such variable, break ties according to the *most constraining variable* variable ordering criterion;
  - if the choice is still not unique, break ties by selecting the variable with the smallest index.
- Value ordering: alphabetical

(a) Provide the variable and value orderings induced by the static ordering strategies.

(b) Provide the search tree that is created by applying naive backtracking on the depicted problem using the static variable and value orderings from the first part of the exercise.

As on the lecture slides, your search tree should be complete and contain all solutions. Depict it in a similar style and discuss its size in comparison to the size of the search tree of the lecture slides.

Exercise 7.2 (1+2 marks)

Consider the 6 queens problem with the partial assignment $\alpha = \{v_1 \mapsto 2, v_2 \mapsto 4\}$:

	v_1	v_2	v_3	v_4	v_5	v_6
1						
2	q					
3						
4		q				
5						
6						

In the following, you may assume that the positions of the two queens that are already on the board are fixed, i.e., that the domain of the corresponding variables contains only the single entry that encodes the depicted position. The domain of the remaining variables contains all 6 possible values, though, which leads to the following domains for all variables:

$$\begin{aligned}
\text{dom}(v_1) &= \{2\}\\
\text{dom}(v_2) &= \{4\}\\
\text{dom}(v_3) &= \{1,2,3,4,5,6\}\\
\text{dom}(v_4) &= \{1,2,3,4,5,6\}\\
\text{dom}(v_5) &= \{1,2,3,4,5,6\}\\
\text{dom}(v_6) &= \{1,2,3,4,5,6\}
\end{aligned}$$

(a) Determine the domains of all variables after applying forward checking in α.

(b) Apply the AC-3 algorithm that has been presented on slide 21 of chapter 25 in the print version of the lecture slides on the constraint network $\mathcal{C}$ with the domains that are the result of (a) until arc consistency is enforced. Select the variables u and v in each iteration of the while loop such that the domain of u changes in the call to `revise`$(\mathcal{C}, u, v)$. Provide u, v, and dom(u) in each iteration. Note that you do *not* have to provide the elements that are inserted into the queue, and you may stop the algorithm as soon as there are no variables u and v such that dom(u) changes.

Exercise 7.3 (1+2 marks)

Consider the constraint network $\mathcal{C} = \langle V, \text{dom}, (R_{uv})\rangle$ with $V = \{A, B, C, D, E, F, G\}$, $\text{dom}(v) = \{1, \ldots, 10\}$ for all $v \in V$ and $R_{uv} = \text{dom}(u) \times \text{dom}(v)$ for all pairs $u, v \in V$ except for:

$$\begin{aligned}
R_{AC} &= \{\langle x, y\rangle \in \text{dom}(A) \times \text{dom}(C) \mid x \leq y\}\\
R_{BG} &= \{\langle x, y\rangle \in \text{dom}(B) \times \text{dom}(G) \mid x < y\}\\
R_{CE} &= \{\langle x, y\rangle \in \text{dom}(C) \times \text{dom}(E) \mid 2x < y\}\\
R_{CG} &= \{\langle x, y\rangle \in \text{dom}(C) \times \text{dom}(G) \mid |x - y| > 3\}\\
R_{DF} &= \{\langle x, y\rangle \in \text{dom}(D) \times \text{dom}(F) \mid |y - x^2| \leq 1\}\\
R_{FG} &= \{\langle x, y\rangle \in \text{dom}(F) \times \text{dom}(G) \mid |x - y| \leq 2\}
\end{aligned}$$

(a) Draw the constraint graph of $\mathcal{C}$.

(b) Apply the algorithm for trees as constraint graphs (slide 12 of chapter 27 in the print version of the lecture slides) to solve $\mathcal{C}$. Provide the following:

- the ordered tree, rooted at A;
- the variable ordering that is given by the ordered tree;
- the sequence of calls of the `revise` method and the resulting variable domains; and
- the result of a backtracking search with a value ordering that prefers smaller values. It suffices to provide the final variable assignments (it is, in particular, not necessary to provide the complete search tree).

Exercise 7.4 (3 marks)

Let $\mathcal{C}$ be a solvable constraint network with acyclic constraint graph. Show that the application of the algorithm for trees as constraint graphs (slide 12 of chapter 27 of the print version of the lecture slides) leads to a solution for $\mathcal{C}$ and that the algorithm does never change the values of assigned variables during application (i.e., the domain of each variable is always non-empty and the partial assignments that are considered are always consistent).

Important: Solutions should be submitted in groups of two students. However, only one student should upload the solution. Please provide both student names on each file and each page you submit. We can only accept a single PDF or a ZIP file containing *.java or *.pddl files and a single PDF.