

Planning and Optimization

G3. Symbolic Search: Uniform-cost and A^* search

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December 18, 2017

Introduction

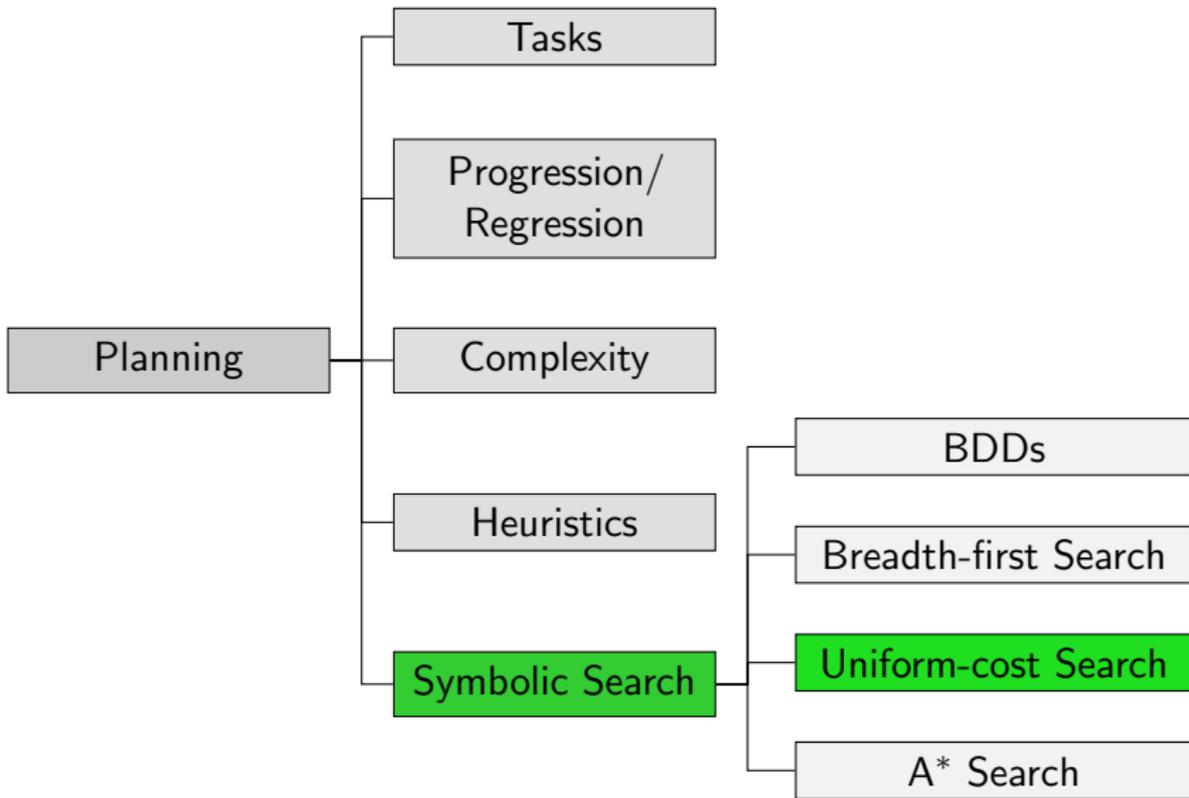
Introduction

- Previous chapter: Symbolic **breadth-first search**
- Optimal plans only guaranteed for unit-cost tasks
(= all operators same cost)
- Optimal planning in explicit-state forward search:
 - (uninformed) uniform-cost search
 - (informed) A* search
 - ...

Analogous algorithms for symbolic (BDD-based) search?

Symbolic Uniform-Cost Search

Content of this Course



Cost-separated Transition Relations

- Previously: **one transition relation** $T_V(O)$ for **all** operators
- Now: **several** transition relations for operators of **same cost**
- Set \mathcal{T} of pairs (T, c) , where T is a transition relation for one/some/all operators of cost c

- All operators must be covered (and nothing else):

$$\bigcup_{(T,c) \in \mathcal{T}} r(T) = r(T_V(O))$$

- The cost must be correct:

$$\text{For } (T, c) \in \mathcal{T}: \text{ if } t \in r(T) \text{ then } t \models \bigvee_{o \in O: \text{cost}(o)=c} T_V(o)$$

Many possibilities to split up $T_V(O)$ ([discussed later](#))

Image Computation

- The apply function (previous chapter) computes the set of states S' that can be reached from a set of states S by applying one operator.
- This is called the **image** of S wrt. transition relation $T_V(O)$.
- Now: image computation for arbitrary transition relations.

```
def image( $B, T$ ):  
     $B := bdd\text{-intersection}(B, T)$   
    for each  $v \in V$ :  
         $B := bdd\text{-forget}(B, v)$   
    for each  $v \in V$ :  
         $B := bdd\text{-rename}(B, v', v)$   
    return  $B$ 
```

Exactly like **apply** but gets transition relation as argument.

Symbolic Uniform-Cost Search (Positive Operator Costs)

```
def symbolic-uniform-cost( $V, I, O, \gamma$ ):  
     $goal := build\text{-}BDD(\gamma)$   
     $\mathcal{T} := make\text{-}transition\text{-}relations(V, O)$   
     $open_0 := bdd\text{-}state(I)$   
    while  $\exists g : open_g \neq \mathbf{0}$ :  
         $g := \min\{g \mid open_g \neq \mathbf{0}\}$   
         $closed_g := open_g$   
        if  $bdd\text{-}intersection(open_g, goal) \neq \mathbf{0}$ :  
            return  $construct\text{-}plan(I, O, goal, closed_*, g)$   
        for all  $(T, c) \in \mathcal{T}$ :  
             $open_{g+c} := bdd\text{-}union(open_{g+c},$   
                                    $image(open_g, T))$   
         $open_g := \mathbf{0}$   
    return unsolvable
```

Pre-image Computation

- The image of S wrt. transition relation T computes the set of states that **can be reached** from S by applying a transition represented by T .
- The **pre-image** of S wrt. T is the set of states **from which we can reach** S by applying a transition represented by T .

```
def pre-image( $B, T$ ):  
  for each  $v \in V$ :  
     $B := \text{bdd-rename}(B, v, v')$   
   $B := \text{bdd-intersection}(B, T)$   
  for each  $v \in V$ :  
     $B := \text{bdd-forget}(B, v')$   
  return  $B$ 
```

Plan Extraction (Positive Operator Costs)

```

def construct-plan( $I, O, goal, closed_*, g$ ):
     $cut := bdd\text{-intersection}(goal, closed_g)$ 
     $init := bdd\text{-state}(I)$ 
     $\pi := \langle \rangle$ 
    while  $bdd\text{-intersection}(cut, init) = \mathbf{0}$ :
        for  $o \in O$ :
             $pre := pre\text{-image}(cut, \tau_V(o))$ 
            if  $c := bdd\text{-intersection}(pre, closed_{g-cost(o)}) \neq \mathbf{0}$ :
                 $cut := c$ 
                 $g := g - cost(o)$ 
                 $\pi := \langle o \rangle \pi$ 
                break
    return  $\pi$ 

```

Zero-cost Operators

What is the problem with zero-cost operators?

- **Search:** could re-open $open_g$ after it was moved to $closed_g$, possibly running into an infinite loop
→ Apply all zero-cost operators before closing
- **Plan extraction:** could loop in zero-cost cycles
→ special treatment

Breadth-first Exploration with Zero-cost Operators

```

def bfs-zero( $B, g, \mathcal{T}, \text{goal}$ ):
     $i := 0$ 
     $\text{closed}_{g,i} := B$ 
    while  $B \neq \mathbf{0}$  and  $\text{bdd-intersection}(B, \text{goal}) = \mathbf{0}$ :
         $B' := \mathbf{0}$ 
        for  $(T, c) \in \mathcal{T}, c = 0$ :
             $B' := \text{bdd-union}(B', \text{image}(B, T))$ 
         $B := \text{bdd-intersection}(B', \text{bdd-complement}(\text{closed}_{g,i}))$ 
         $i := i + 1$ 
         $\text{closed}_{g,i} := \text{bdd-union}(B, \text{closed}_{g,i-1})$ 
    return  $\text{closed}_{g,i}$ 

```

Symbolic Uniform-Cost Search

```

def symbolic-uniform-cost( $V, I, O, \gamma$ ):
     $goal := build\text{-}BDD(\gamma)$ 
     $\mathcal{T} := make\text{-}transition\text{-}relations(V, O)$ 
     $open_0 := bdd\text{-}state(I)$ 
    while  $\exists g : open_g \neq \mathbf{0}$ :
         $g := \min\{g \mid open_g \neq \mathbf{0}\}$ 
         $open_g := bfs\text{-}zero(open_g, g, \mathcal{T}, goal)$ 
         $closed_g := open_g$ 
        if  $bdd\text{-}intersection(open_g, goal) \neq \mathbf{0}$ :
            return  $construct\text{-}plan(I, O, goal, closed_*, g)$ 
        for all  $(T, c) \in \mathcal{T}$  with  $c > 0$ :
             $open_{g+c} := bdd\text{-}union(open_{g+c},$ 
                                    $image(open_g, T))$ 
         $open_g := \mathbf{0}$ 
    return unsolvable
  
```

Plan Extraction with Zero-cost Operators

Needs all closed sets form bfs-zero and symbolic-uniform-cost.

```

def construct-plan( $I$ ,  $O$ , goal,  $closed_{*,*}$ ,  $g$ ):
  cut := bdd-intersection(goal,  $closed_g$ )
  init := bdd-state( $I$ );  $\pi := \langle \rangle$ 
  while bdd-intersection(cut, init) = 0:
    cut,  $\pi :=$  get-to-bfs-level-0(cut,  $g$ ,  $closed_{g,*}$ ,  $\pi$ ,  $O$ )
    if  $g = 0$ :
      return  $\pi$ 
    for  $o \in O$  with  $cost(o) > 0$ :
      pre := pre-image(cut,  $\tau_V(o)$ )
      if  $c :=$  bdd-intersection(pre,  $closed_{g-cost(o)}) \neq 0$ :
        cut :=  $c$ ;  $\pi := \langle o \rangle \pi$ 
         $g := g - cost(o)$ 
        break
  return  $\pi$ 

```

Plan Extraction: Zero-Cost Plan Fragment

```

def get-to-bfs-level-0(cut, g, closedg,*,  $\pi$ , O):
    level := 0
    while bdd-intersection(cut, closedg,level) = 0:
        level := level + 1
    while level  $\neq$  0:
        for o  $\in$  O with cost(o) = 0:
            pre := pre-image(cut,  $\tau_V(o)$ )
            if c := bdd-intersection(pre, closedg,level-1)  $\neq$  0:
                cut := c
                level := level - 1
                 $\pi$  :=  $\langle o \rangle \pi$ 
                break
    return cut,  $\pi$ 
  
```

Pruning of Closed States

- In explicit-state uniform-cost search, we never re-expand closed states.
- We can easily introduce such pruning in symbolic uniform-cost search.

Uniform-Cost Search with Pruning of Closed States

```

def symbolic-uniform-cost( $V, I, O, \gamma$ ):
     $goal := build\text{-}BDD(\gamma)$ 
     $\mathcal{T} := make\text{-}transition\text{-}relations(V, O)$ 
     $open_0 := bdd\text{-}state(I)$ 
    while  $\exists g : open_g \neq \mathbf{0}$ :
         $g := \min\{g \mid open_g \neq \mathbf{0}\}$ 
         $open_g := bfs\text{-}zero(open_g, g, \mathcal{T}, goal, closed_*)$ 
         $closed_g := open_g$ 
        if  $bdd\text{-}intersection(open_g, goal) \neq \mathbf{0}$ :
            return  $construct\text{-}plan(I, O, goal, closed_*, g)$ 
        for all  $(T, c) \in \mathcal{T}$  with  $c > 0$ :
             $open_{g+c} := bdd\text{-}union(open_{g+c},$ 
                                    $image(open_g, T))$ 

     $open_g := \mathbf{0}$ 
    return unsolvable
  
```

bfs-zero with Pruning of Closed States

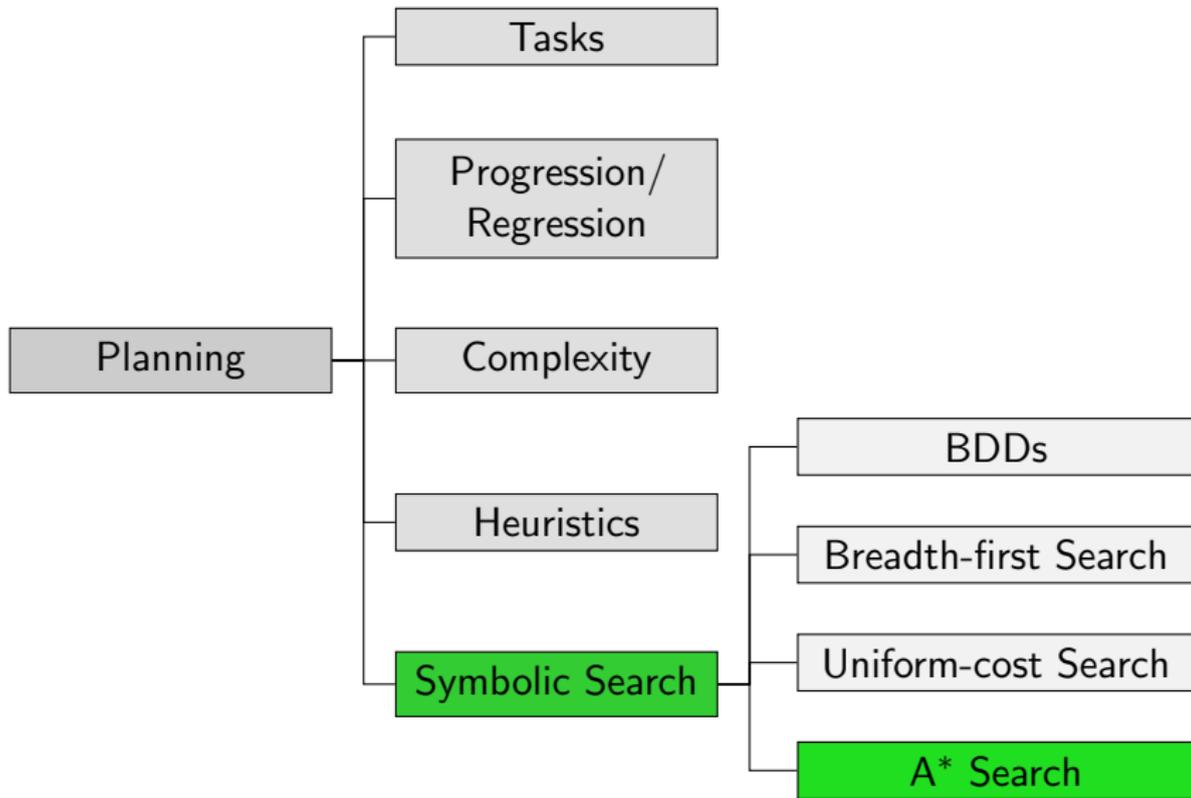
```

def bfs-zero( $B, g, \mathcal{T}, \text{goal}, \text{prune}$ ):
    for  $P \in \text{prune}$ :
         $B := \text{bdd-intersection}(B, \text{bdd-complement}(P))$ 
     $i := 0$ 
     $\text{closed}_{g,i} := B$ 
    while  $B \neq \mathbf{0}$  and  $\text{bdd-intersection}(B, \text{goal}) = \mathbf{0}$ :
         $B' := \mathbf{0}$ 
        for  $(T, c) \in \mathcal{T}, c = 0$ :
             $B' := \text{bdd-union}(B', \text{image}(B, T))$ 
         $B := \text{bdd-intersection}(B', \text{bdd-complement}(\text{closed}_{g,i}))$ 
        for  $P \in \text{prune}$ :
             $B := \text{bdd-intersection}(B, \text{bdd-complement}(P))$ 
         $i := i + 1$ 
         $\text{closed}_{g,i} := \text{bdd-union}(B, \text{closed}_{g,i-1})$ 
    return  $\text{closed}_{g,i}$ 

```

Symbolic A*

Content of this Course



Symbolic A*

- Difference between explicit-state uniform-cost search and A*:
heuristic to guide search
- $f = g + h$
- Analogously in symbolic search
- Heuristic given as set $heur$ of BDDs $heur_h$
for each heuristic estimate h

Symbolic A* (with Consistent Heuristic)

```

def symbolic-AStar( $V, I, O, \gamma, \text{heur}$ ):
     $\text{goal} := \text{build-BDD}(\gamma)$ 
     $\mathcal{T} := \text{make-transition-relations}(V, O)$ 
     $\text{open}_{0,h(I)} := \text{bdd-state}(I)$ 
    while  $\exists g, h : \text{open}_{g,h} \neq \mathbf{0}$ :
         $f := \min\{f \mid \exists g, h : \text{open}_{g,h} \neq \mathbf{0}, f = g + h\}$ 
         $g := \min\{g \mid \exists h : \text{open}_{g,h} \neq \mathbf{0}, f = g + h\}$ 
         $\text{open}_{g,*} := \text{expand}_0(\text{open}_{*,*}, g, h, \mathcal{T}, \text{goal}, \text{heur}, \text{closed}_*)$ 
         $\text{closed}_g := \text{bdd-union}(\text{closed}_g, \text{open}_{g,h})$ 
        if  $\text{bdd-intersection}(\text{open}_{g,h}, \text{goal}) \neq \mathbf{0}$ :
            return  $\text{construct-plan}(I, O, \text{goal}, \text{closed}_*, g)$ 
         $\text{open}_{*,*} := \text{expand}_{>0}(\text{open}_{*,*}, g, h, \mathcal{T}, \text{heur})$ 
         $\text{open}_{g,h} := \mathbf{0}$ 
    return unsolvable
  
```

Symbolic A* (with Consistent Heuristic)

```

def symbolic-AStar( $V, I, O, \gamma, \text{heur}$ ):
     $\text{goal} := \text{build-BDD}(\gamma)$ 
     $\mathcal{T} := \text{make-transition-relations}(V, O)$ 
     $\text{open}_{0,h(I)} := \text{bdd-state}(I)$ 
    while  $\exists g, h : \text{open}_{g,h} \neq \mathbf{0}$ :
         $f := \min\{f \mid \exists g, h : \text{open}_{g,h} \neq \mathbf{0}, f = g + h\}$ 
         $g := \min\{g \mid \exists h : \text{open}_{g,h} \neq \mathbf{0}, f = g + h\}$ 
         $\text{open}_{g,*} := \text{expand}_0(\text{open}_{*,*}, g, h, \mathcal{T}, \text{goal}, \text{heur}, \text{closed}_*)$ 
         $\text{closed}_g := \text{bdd-union}(\text{closed}_g, \text{open}_{g,h})$ 
        if  $\text{bdd-intersection}(\text{open}_{g,h}, \text{goal}) \neq \mathbf{0}$ :
            return  $\text{construct-plan}(I, O, \text{goal}, \text{closed}_*, g)$ 
         $\text{open}_{*,*} := \text{expand}_{>0}(\text{open}_{*,*}, g, h, \mathcal{T}, \text{heur})$ 
         $\text{open}_{g,h} := \mathbf{0}$ 
    return unsolvable
  
```

For performance it is important to expand the **minimum** g value.

Expand States and Update Open Lists

```
def expand0(open*,*, g, h,  $\mathcal{T}$ , goal, heur, prune):  
    open-zero := bfs-zero(openg,h, (g, h),  $\mathcal{T}$ , goal, prune)  
    for heurh' ∈ heur, h ≤ h' < ∞:  
        B' := bdd-intersection(heurh', open-zero)  
        openg,h' := bdd-union(openg,h', B')  
    return openg,*
```

```
def expand>0(open*,*, g, h,  $\mathcal{T}$ , heur):  
    for all (T, c) ∈  $\mathcal{T}$ , c > 0:  
        successors := image(openg,h, T)  
        for heurh' ∈ heur, h - c ≤ h' < ∞:  
            B' := bdd-intersection(heurh', successors)  
            openg+c,h' := bdd-union(openg+c,h', B')  
    return open*,*
```

Heuristics

How can we generate symbolic heuristics?

- **Symbolic Pattern Databases**
 - Uniform-cost search can easily be adapted to regression search.
 - Can search backwards in abstract transition systems
 - BDD for closed states with (backwards-) g -value i is heuristic BDD for $h = i$.
- **Merge-and-Shrink Abstractions**
 - **Algebraic Decision Diagrams** are like BDDs but sink nodes are labeled with arbitrary numbers.
 - Can map states to numbers.
 - Cascading tables of merge-and-shrink heuristics with linear merge strategy can efficiently be transformed into an ADD.
 - Result can be used in symbolic search instead of BDD set.

Discussion

Importance of Variable Ordering

- For good performance, we need a **good variable ordering**.
 - Variables that refer to the same state variable before and after operator application (v and v') should be **neighbors** in the transition relation BDD.
- This is important for the performance of *BDD-rename* in the *image* and *pre-image* computation.

Transition Relations in \mathcal{T}

- We only required that all operators are represented by some $(T, c) \in \mathcal{T}$ and that the costs are correct.
- Extreme cases:
 - One element $(\tau_V(o), cost(o))$ for each operator o
 - Only one element for each operator cost, covering all operators of that cost.
- Trade-off:
 - Large number of entries leads to large number of image computations.
 - Size of T can grow exponentially with number of covered operators.
- There exist different aggregation strategies.

Performance

- In symbolic planning, blind search is often better than informed search.
- Practical implementations also perform **regression** or **bidirectional** search.
- This is only a minor modification of uniform-cost search.

Summary

Summary

- **Symbolic search** operates on **sets of states** instead of individual states as in explicit-state search.
- State sets and transition relations can be represented as **BDDs**.
- A good variable ordering and an efficient image computation are crucial for performance.

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