

# Planning and Optimization

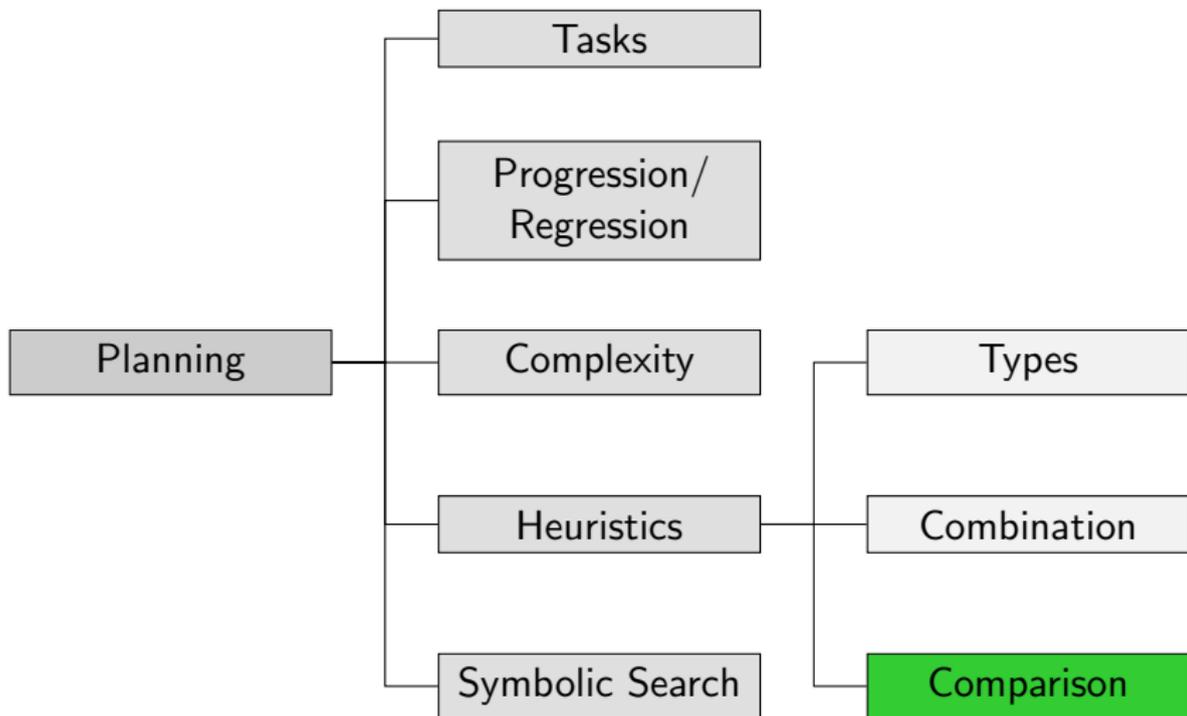
## F6. Comparison of Heuristic Families II

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# Content of this Course



# Landmarks vs. $h^{\max}$

Landmarks to  $h^{\max}$ 

## Theorem

*Elementary landmark heuristics can be compiled into additive  $h^{\max}$  heuristics in polynomial time.*

## Proof.

Let  $L$  be a subset of the operators. If  $L^+$  is not a landmark for  $s$  in  $\Pi^+$  then  $h_L(s) = 0$  and therefore trivially  $h^{\max}(s) \geq h_L(s)$ .

Otherwise no goal state of  $\Pi^+$  is reachable from  $s$  without an operator from  $L^+$ . So if  $h^{\max}(s) \neq \infty$  then the cost computation of  $h^{\max}$  must use an operator from  $L^+$  and therefore  $h^{\max}(s) \geq \min_{o \in L} \text{cost}(o) = h_L$ . □

# $h^{\max}$ to Landmarks

## Theorem

*For states with finite  $h^{\max}$  value, the  $h^{\max}$  heuristic can be compiled into additive elementary landmark heuristics in polynomial time.*

## Proof sketch:

The LM-Cut heuristic computes in each step a cut landmark and adapts the operator costs. Let  $cost_i, cost_{i+1}$  be the operator costs before and after an iteration that discovered landmark  $L$ . Then  $h_{cost_i}^{\max}(s) \leq h_{L, cost_i}(s) + h_{cost_{i+1}}^{\max}(s)$ . The core argument is that every “reasonable” path in the justification graph enters the goal zone only once and therefore uses only one operator from  $L$ . So reducing the cost of each operator in  $L$  by  $h_{L, cost_i}(s)$  cannot reduce  $h^{\max}$  by more than this value. The overall result of the theorem follows from a recursive application of the proof while  $h_{cost_{i+1}}^{\max}(s) > 0$ .  $\square$

# Abstractions vs. Critical Path

# $h^m$ to PDBs

## Theorem

*There is no polynomial-time compilation from  $h^m$  heuristics into additive PDB heuristics.*

## Proof.

We know that elementary landmarks are in polynomial time compilable into additive  $h^{\max}$  but not into additive PDB heuristics. So there is no polynomial-time compilation from  $h^{\max} = h^1$  into additive PDB heuristics.

As  $h^m \geq h^1$  for  $m \geq 1$ , this holds for any  $m$ . □

PDBs to  $h^m$ 

## Theorem

*There is no polynomial-time compilation of PDB heuristics into additive  $h^m$  heuristics.*

## Proof.

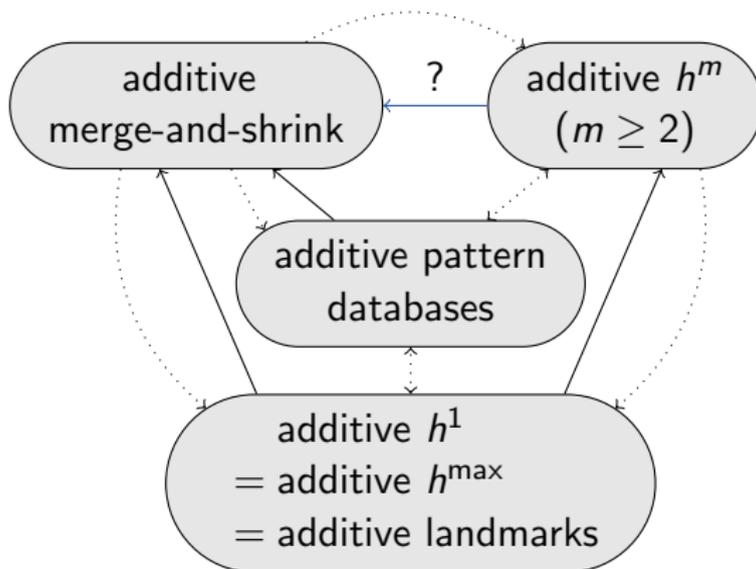
Consider family  $(\Pi_n)_{n \in \mathbb{N}_1}$  of STRIPS tasks, where  $\Pi_n = \langle V_n, I_n, O_n, \gamma \rangle$  with  $V_n = \{v_1, \dots, v_n\}$ ,  $I_n(v) = \mathbf{F}$  for  $v \in V_n$ ,  $O = \{ \langle \bigwedge_{i=1}^{j-1} v_i, v_j \wedge \bigwedge_{i=1}^{j-1} \neg v_i, 1 \rangle \mid 1 \leq j \leq n \}$  and  $\gamma = \bigwedge_{i: i\text{-th bit in } \text{bin}(n) \text{ is } 1} v_i$ .

A PDB on pattern  $\{v_1, \dots, v_{\lceil \log n \rceil}\}$  has  $O(n)$  states and encodes the perfect goal distance  $h^*(I) = n$ .

For a perfect initial estimate, the  $h^m$  heuristic needs to consider variable subsets up to size  $\lceil \log n \rceil$ . As  $m$  must be fixed due to the polynomial-time requirement, we can thus find for any such  $m$  a large enough  $n$  that proves the theorem. □

# Overview

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Solid arc: poly-time compilation exists

Dotted arc: compilation not possible

# What else?

## Post-hoc optimization

- For PDBs it computes state-specific additive set of PDB heuristics. → Covered by results on PDB heuristics.
- Analogously for other classes of heuristics.

## Potential heuristics

- → Exercises

# Missing Results

So far no results for

- landmarks not based on delete relaxation ( $\Pi^m$  landmarks),
- flow heuristics, and
- compilability from  $h^m$  heuristics into additive merge-and-shrink heuristics.

# Summary

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- Relaxation-based landmark heuristics dominate additive  $h^{\max}$  heuristics and vice versa.
- Additive critical path heuristics with  $m \geq 2$  strictly dominate relaxation-based landmark heuristics and additive  $h^{\max}$  heuristics.
- Merge-and-shrink heuristics strictly dominate relaxation-based landmark heuristics and additive  $h^{\max}$  heuristics.
- PDB heuristics are incomparable with relaxation-based landmark heuristics and additive  $h^{\max}$  heuristics.

# Literature



Malte Helmert and Carmel Domshlak.

Landmarks, Critical Paths and Abstractions: What's the Difference Anyway?

*Proc. ICAPS 2009*, pp. 162–169, 2009.