Planning and Optimization

F4. Potential Heuristics & Connections

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December 6, 2017

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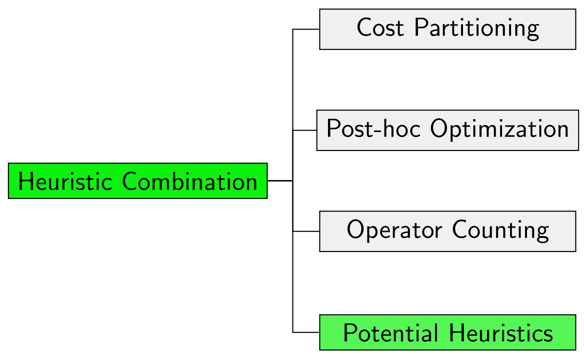
F4.1 Potential Heuristics

Content of this Course: Heuristic Combination



Motivation

- Operator-counting heuristics solve an LP to compute the heuristic estimate for a single state.
- Can we also define an entire heuristic function solving only one LP?
- Axiomatic approach for defining heuristics:
  - What should a heuristic look like mathematically?
  - Which properties should it have?
- Define a space of interesting heuristics.
- Use optimization to pick a good representative.

Potential Heuristics

Potential Heuristics: Idea

Heuristic design as an optimization problem:

- Define simple numerical state features $f_1, \dots, f_n$.
- Consider heuristics that are linear combinations of features:

$$h(s) = w_1 f_1(s) + \cdots + w_n f_n(s)$$

  with weights (potentials) $w_i \in \mathbb{R}$
- Find potentials for which h is admissible and well-informed.

Motivation:

- declarative approach to heuristic design
- heuristic very fast to compute if features are

Features

Definition (feature)

A (state) feature for a planning task is a numerical function defined on the states of the task: $f : S \to \mathbb{R}$.

Potential Heuristics

Definition (potential heuristic)

A potential heuristic for a set of features $\mathcal{F} = \{f_1, \dots, f_n\}$ is a heuristic function h defined as a linear combination of the features:

$$h(s) = w_1 f_1(s) + \cdots + w_n f_n(s)$$

with weights (potentials) $w_i \in \mathbb{R}$.

$\rightsquigarrow$ cf. evaluation functions for board games like chess

Atomic Potential Heuristics

Atomic features test if some atom is true in a state:

Definition (atomic feature)

Let $X = x$ be an atom of a FDR planning task.

The atomic feature $f_{X=x}$ is defined as:

$$f_{X=x}(s) = \begin{cases} 1 & \text{if variable } X \text{ has value } x \text{ in state } s \\ 0 & \text{otherwise} \end{cases}$$

- We only consider atomic potential heuristics, which are based on the set of all atomic features.
- Example for a task with state variables X and Y:

$$h(s) = 3f_{X=a} + \tfrac{1}{2}f_{X=b} - 2f_{X=c} + \tfrac{5}{2}f_{Y=d}$$

How to Set the Weights?

We want to find good atomic potential heuristics:

- admissible
- consistent
- well-informed

How to achieve this? Linear programming to the rescue!

Admissible and Consistent Potential Heuristics

Constraints on potentials characterize (= are necessary and sufficient for) admissible and consistent atomic potential heuristics:

Goal-awareness

$$\sum_{\text{goal atoms } a} w_a = 0$$

Consistency

$$\sum_{\substack{a \text{ consumed} \\ \text{by } o}} w_a - \sum_{\substack{a \text{ produced} \\ \text{by } o}} w_a \leq cost(o) \quad \text{for all operators } o$$

Remarks:

- assumes transition normal form (not a limitation)
- goal-aware and consistent = admissible and consistent

Well-Informed Potential Heuristics

How to find a well-informed potential heuristic?

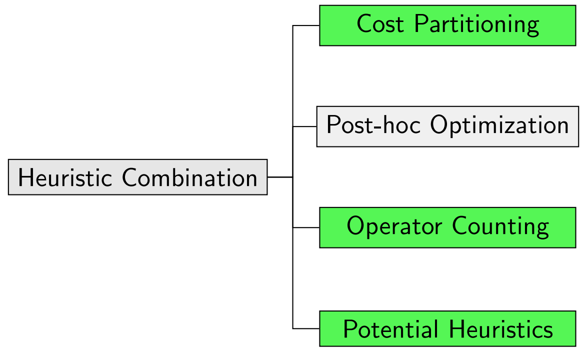
$\rightsquigarrow$ encode quality metric in the objective function and use LP solver to find a heuristic maximizing it

Examples:

- maximize heuristic value of a given state (e.g., initial state)
- maximize average heuristic value of all states (including unreachable ones)
- maximize average heuristic value of some sample states
- minimize estimated search effort

F4.2 Connections

Content of this Course: Heuristic Combination



Potential and Flow Heuristic

Theorem

For state s, let $h^{\mathrm{maxpot}}(s)$ denote the maximal heuristic value of all admissible and consistent atomic potential heuristics in s.

Then $h^{\mathrm{maxpot}}(s) = h^{\mathit{flow}}(s)$.

Proof idea: compare dual of $h^{\mathrm{flow}}(s)$ LP to potential heuristic constraints optimized for state s.

If we optimize the potentials for a given state then for this state it equals the flow heuristic.

Operator Counting and General Cost Partitioning

Theorem

Combining operator-counting heuristics in one LP
is equivalent to
computing their optimal general cost partitioning.

Proof idea: The linear programs are each others duals.

Use the Theorem to Combine Heuristics

- Easy way to compute cost partitioning of heuristics
  - LP can be more compact (variable elimination)
  - No need for one variable per operator and subproblem
- Even better combination of heuristics with IP heuristic
  - Considers that operator cannot be used 1.5 times
  - But computation is no longer polynomial

Use the Theorem to Analyze Heuristics

Analyze operator counting heuristics
Example: flow heuristic

1. Group linear constraints into sets of operator-counting constraints
   - One group of flow constraints per variable
2. Figure out what heuristic is computed with just one such set
   - Minimizing total cost while respecting flow in projection to one variable
   - Shortest path in projection
3. Your original operator-counting heuristic computes the optimal general cost partition of those component heuristics
   - Flow heuristic = gOCP(atomic projection heuristics)

Other Examples

What about the rest of our examples?

- Landmark constraints
  - gOCP(individual landmark heuristics)
- Post-hoc optimization heuristic
  - gOCP(heuristics that spend a minimum cost on relevant ops)
  - Also: cost partitioning over atomic projection heuristics
    - Operator costs not independent
    - Scale with one factor per projection

F4.3 Summary

Summary

- The combination into one operator-counting heuristic corresponds to the computation of the optimal general cost partitioning for the ingredient heuristics.
- General cost partitioning, operator-counting constraints and potential heuristics are facets of the same phenomenon.
- Study of each reinforces understanding of the others.
- Potential heuristics can be used as fast admissible approximations of h^{flow}.
- Generalization beyond h^{flow}: use non-atomic features
- If features are cheap to compute, the heuristic evaluation for every state is extremely fast.

Literature

Florian Pommerening, Malte Helmert, Gabriele Röger and Jendrik Seipp.
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