Planning and Optimization

F2. Cost Partitioning: Landmarks and Generalization

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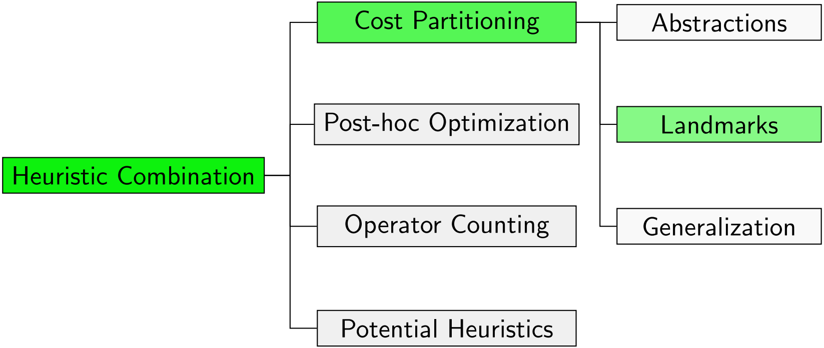
Cost Partitioning for Landmarks

Content of this Course: Heuristic Combination



Reminder: Disjunctive Action Landmarks

Disjunctive action landmark

- Set of operators
- Every plan uses at least one of them
- Landmark cost = cost of cheapest operator

Reminder: Cost Partitioning Heuristic for Landmarks

We have already seen a landmark heuristic based on cost partitioning:

Definition (Uniform Cost Partitioning Heuristic for Landmarks)

Let $\mathcal{L}$ be a set of disjunctive action landmarks.

The uniform cost partitioning heuristic $h^{\mathrm{UCP}}(\mathcal{L})$ is defined as

$$h^{\mathrm{UCP}}(\mathcal{L}) = \sum_{L \in \mathcal{L}} \min_{o \in L} c'(o) \text{ with}$$

$$c'(o) = cost(o)/|\{L \in \mathcal{L} \mid o \in L\}|.$$

Reminder: Proof Back Then

Theorem (Uniform Cost Partitioning Heuristic is Admissible)

Let $\mathcal{L}$ be a set of disjunctive action landmarks for state s of Π. Then $h^{UCP}(\mathcal{L})$ is an admissible heuristic estimate for s.

Proof.

Let $\pi = \langle o_1, \ldots, o_n \rangle$ be an optimal plan for s. For $L \in \mathcal{L}$ define a new cost function $cost_L$ as $cost_L(o) = c'(o)$ if $o \in L$ and $cost_L(o) = 0$ otherwise. Let Π_L be a modified version of Π, where for all operators o the cost is replaced with $cost_L(o)$.
$(\ldots)$ □

$$\sum_{L \in \mathcal{L}} cost_L(o) = \sum_{L \in \mathcal{L}: o \in L} cost(o) / |\{L \in \mathcal{L} \mid o \in L\}| = cost(o)$$

Heuristic is Based on Cost Partitioning

- For disj. action landmark L of state s in task Π', let $h_{L,\Pi'}(s)$ be the cost of L in Π'. Then $h_{L,\Pi'}(s)$ is admissible.
- Consider set $\{L_1, \dots, L_n\}$ of disj. action landmarks for state s of task Π.
- Use cost partitioning $\langle cost_{L_1}, \dots, cost_{L_n} \rangle$, where

$$cost_{L_i}(o) = \begin{cases} cost(o)/|\{L \in \mathcal{L} \mid o \in L\}| & \text{if } o \in L_i \\ 0 & \text{otherwise} \end{cases}$$

- Let $\langle \Pi_{L_1}, \dots, \Pi_{L_n} \rangle$ be the tuple of induced tasks.
- $h(s) = \sum_{i=1}^{n} h_{L_i,\Pi_{L_i}}(s)$ is an admissible estimate for s in Π.
- h is uniform cost partitioning heuristic for landmarks.

Optimal Cost Partitioning for Landmarks

Can we find a better cost partitioning?

- Use again LP that covers heuristic computation and cost partitioning.
- LP variable Cost_L for cost of landmark L in induced task (corresponds to $h_{L_i,\Pi_{L_i}}$)
- Explicit variables for cost partitioning not necessary. Use implicitly $cost_L(o) = \text{Cost}_L$ for all $o \in L$ and 0 otherwise.

Optimal Cost Partitioning for Landmarks: LP

Variables

Cost_L for each disj. action landmark $L \in \mathcal{L}$

Objective

Maximize $\sum_{L \in \mathcal{L}} \text{Cost}_L$

Subject to

$$\sum_{L \in \mathcal{L} : o \in L} \text{Cost}_L \leq cost(o) \quad \text{for all operators } o$$

$$\text{Cost}_L \geq 0 \quad \text{for all landmarks } L \in \mathcal{L}$$

Optimal Cost Partitioning for Landmarks (Dual view)

Variables

Count_o for each operator o

Objective

Minimize $\sum_o \text{Count}_o \cdot cost(o)$

Subject to

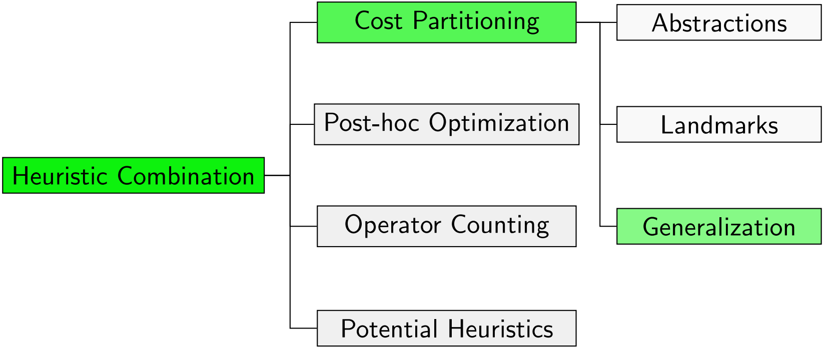
$$\sum_{o \in L} \text{Count}_o \geq 1 \text{ for all landmarks } L$$

$$\text{Count}_o \geq 0 \text{ for all operators } o$$

Minimize "plan cost" with all landmarks satisfied.

General Cost Partitioning

Content of this Course: Heuristic Combination



General Cost Partitioning

Cost functions usually non-negative

- We tacitly also required this for task copies
- Makes intuitively sense: original costs are non-negative
- But: not necessary for cost-partitioning!

General Cost Partitioning

Definition (General Cost Partitioning)

Let Π be a planning task with operators O.

A general cost partitioning for Π is a tuple $\langle cost_1, \dots, cost_n \rangle$, where

- $cost_i : O \to \mathbb{R}$ for $1 \leq i \leq n$ and
- $\sum_{i=1}^{n} cost_i(o) \leq cost(o)$ for all $o \in O$.

General Cost Partitioning: Admissibility

Theorem (Sum of Solution Costs is Admissible)

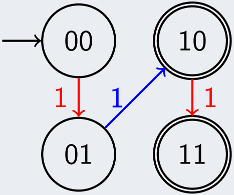
Let Π be a planning task, $\langle cost_1, \dots, cost_n \rangle$ be a general cost partitioning and $\langle \Pi_1, \dots, \Pi_n \rangle$ be the tuple of induced tasks.

Then the sum of the solution costs of the induced tasks is an admissible heuristic for Π, i.e., $\sum_{i=1}^{n} h^*_{\Pi_i} \leq h^*_{\Pi}$.

(Proof omitted.)

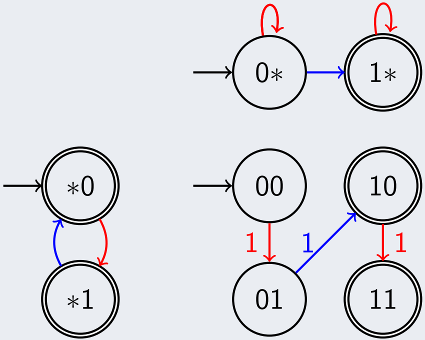
General Cost Partitioning: Example

Example



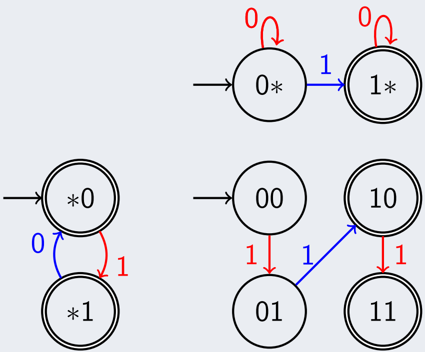
General Cost Partitioning: Example

Example



General Cost Partitioning: Example

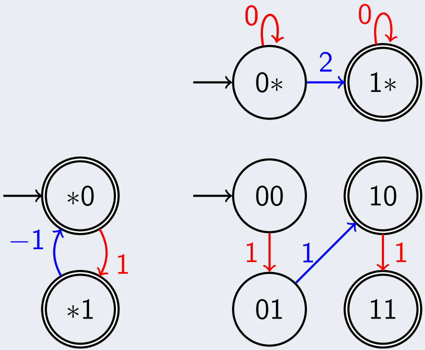
Example



Heuristic value: $0 + 1 = 1$

General Cost Partitioning: Example

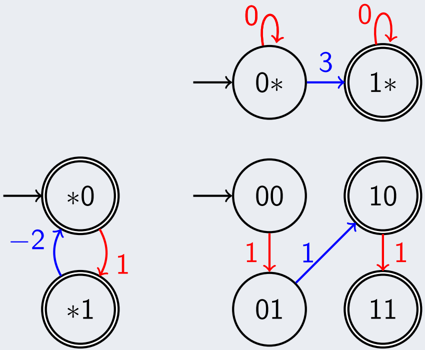
Example



Heuristic value: $0 + 2 = 2$

General Cost Partitioning: Example

Example



Heuristic value: $-\infty + 3 = -\infty$

LP for Shortest Path in State Space with Negative Costs

Variables

Distance_s for each state s,
GoalDist

Objective

Maximize GoalDist

Subject to

$\text{Distance}_{s_I} = 0$	for the initial state s_I
$\text{Distance}_{s'} \leq \text{Distance}_s + cost(o)$	for all alive transitions $s \xrightarrow{o} s'$
$\text{GoalDist} \leq \text{Distance}_{s_\star}$	for all goal states $s_\star$

alive: on any path from initial state to goal state

Modification also correct (but unnecessary) for non-negative costs

General Cost Partitioning: Remarks

- More powerful than non-negative cost partitioning
- Optimal general cost partitioning:
  omit constraints to non-negative cost variables
  - optimal cost partitioning maximizes objective value
  - removing constraints can only increase heuristic value
- Optimal general cost partitioning is never worse than an optimal non-negative cost partitioning.

Summary

Summary

- We can compute an optimal cost partitioning for a given set of disjunctive action landmarks in polynomial time.
- In constrast to standard (non-negative) cost partitioning, general cost partitioning allows negative operators costs.
- General cost partitioning has the same relevant properties as non-negative cost partitioning but is more powerful.

Literature (1)

References on cost partitioning:

Stefan Edelkamp.
Automated Creation of Pattern Database Search Heuristic.
Proc. MoChArt 2006, pp. 35–50, 2006.
Introduces 0-1 cost partitioning for abstraction heuristics.

Michael Katz and Carmel Domshlak.
Structural Patterns Heuristics: Basic Idea and Concrete Instance.
ICAPS 2007 HDIP Workshop, 2007.
Introduces arbitrary non-negative cost partitioning.

Literature (2)

- Fan Yang, Joseph C. Culberson, Robert Holte, Uzi Zahavi and Ariel Felner.
  A General Theory of Additive State Space Abstractions.
  JAIR 32, pp. 631–662, 2008.
  Introduces arbitrary non-negative cost partitioning.

- Erez Karpas and Carmel Domshlak.
  Cost-optimal Planning with Landmarks.
  Proc. IJCAI 2009, pp. 1728–1733, 2009.
  Introduces optimal cost partitioning for landmarks (different formulation than in the slides).

Literature (3)

Emil Keyder, Silvia Richter and Malte Helmert.
Sound and Complete Landmarks for And/Or Graphs.
Proc. ECAI 2010, pp. 335–340, 2010.
Smaller LP for optimal cost partitioning for landmarks.

Blai Bonet and Malte Helmert.
Strengthening Landmark Heuristics via Hitting Set.
Proc. ECAI 2010, pp. 329–334, 2010.
Dual LP for optimal cost partitioning for landmarks.

Literature (4)

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Artificial Intelligence 174 (12–13), pp. 767–798, 2010.
Introduces optimal cost partitioning for abstraction heuristics.

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From Non-Negative to General Operator Cost Partitioning.
Proc. AAAI 2015, pp. 3335–3341, 2015.
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