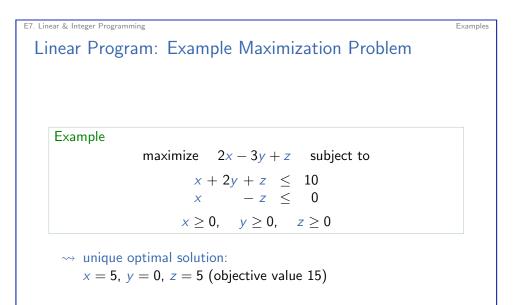


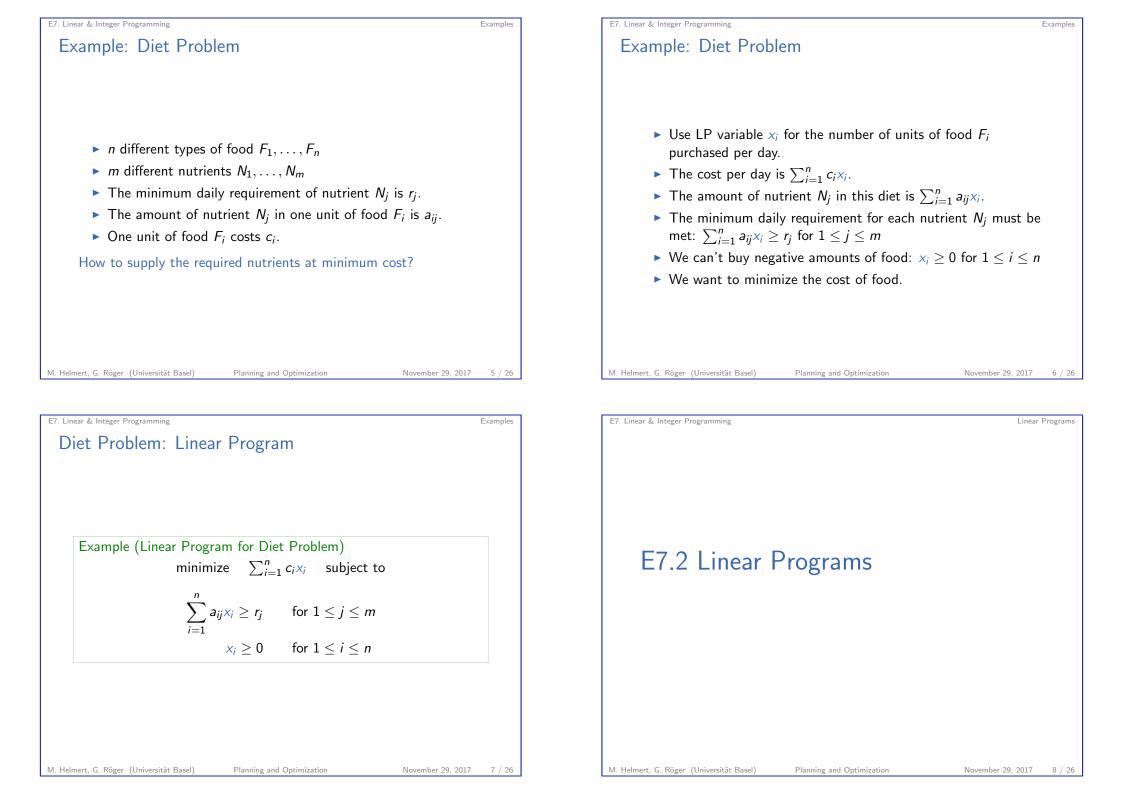
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E7.1 Examples			
E7.2 Linear Progr	ams		
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Linear Programs and Integer Programs

Linear Program

- A linear program (LP) consists of:
 - ► a finite set of real-valued variables V
 - ▶ a finite set of linear inequalities (constraints) over V
 - \blacktriangleright an objective function, which is a linear combination of V
 - which should be minimized or maximized.

Integer program (IP): ditto, but with some integer-valued variables

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E7. Linear & Integer Programming



A standard maximum problem is often given by

- an *m*-vector $\mathbf{b} = \langle b_1, \ldots, b_m \rangle^T$,
- an *n*-vector $\mathbf{c} = \langle c_1, \ldots, c_n \rangle^T$,
- \blacktriangleright and an $m \times n$ matrix

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12}x_2 & \dots & a_{1n} \\ a_{21} & a_{22}x_2 & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$

• Then the problem is to find a vector $\mathbf{x} = \langle x_1, \dots, x_n \rangle^T$ to maximize $\mathbf{c}^T \mathbf{x}$ subject to $\mathbf{A} \mathbf{x} \leq \mathbf{b}$ and $\mathbf{x} \geq \mathbf{0}$.

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Linear Programs

Standard Maximum Problem

Normal form for maximization problems:

Definition (Standard Maximum Problem) Find values for x_1, \ldots, x_n , to maximize

$$c_1x_1+c_2x_2+\cdots+c_nx_n$$

subject to the constraints

 $a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \le b_1$ $a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \le b_2$ \vdots

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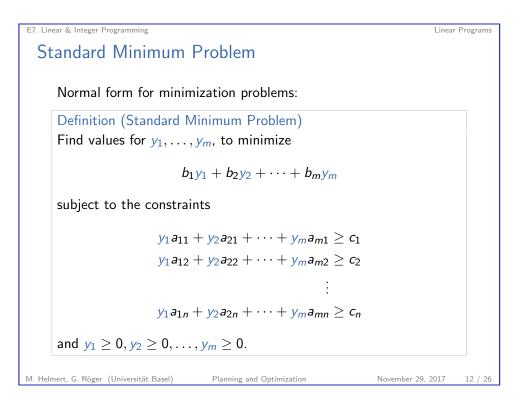
 $a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \leq b_m$

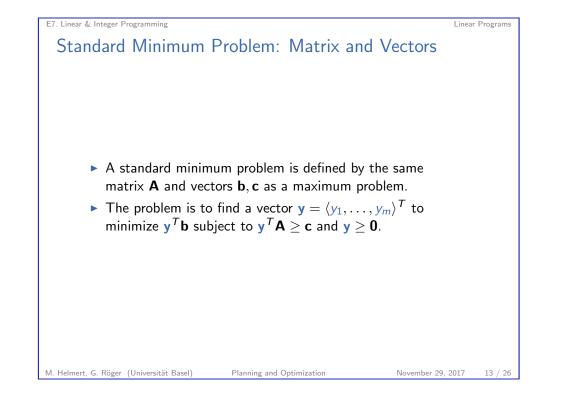
and $x_1 \ge 0, x_2 \ge 0, \dots, x_n \ge 0$.

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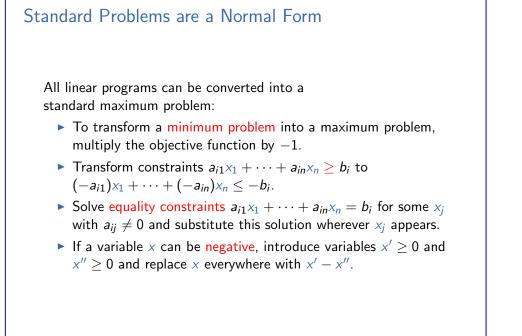
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Linear Programs





E7. Linear & Integer Programming



	7.	Linear	&	Integer	Programming
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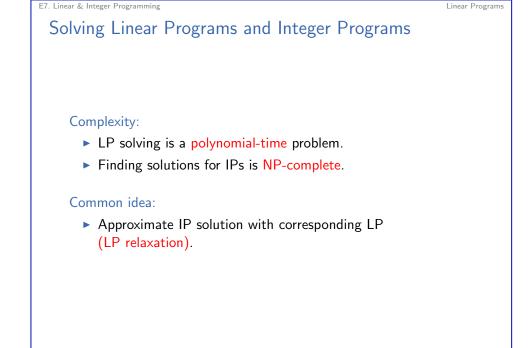
Terminology

- A vector x for a maximum problem or y for a minimum problem is feasible if it satisfies the constraints.
- A linear program is feasible if there is such a feasible vector.
 Otherwise it is infeasible.
- A feasible maximum (resp. minimum) problem is unbounded if the objective function can assume arbitrarily large positive (resp. negative) values at feasible vectors. Otherwise it it bounded.
- The objective value of a bounded feasible maximum (resp. minimum) problem is the maximum (resp. minimum) value of the objective function with a feasible vector.

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LP Relaxation

Theorem (LP Relaxation)

The LP relaxation of an integer program is the problem that arises by removing the requirement that variables are integer-valued.

For a maximization (resp. minimization) problem, the objective value of the LP relaxation is an upper (resp. lower) bound on the value of the IP.

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Proof idea.

Every feasible vector for the IP is also feasible for the LP.

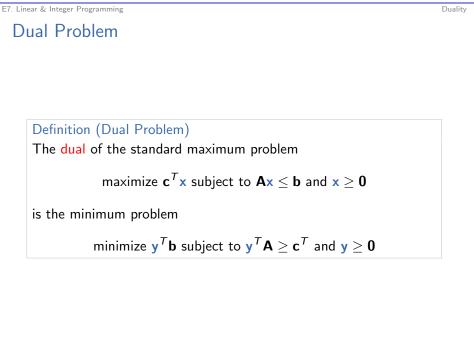
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E7. Linear & Integer Programming Some LP Theory: Duality

Some LP theory: Every LP has an alternative view (its dual).

- roughly: variables and constraints swap roles
- dual of maximization LP is minimization LP and vice versa
- dual of dual: original LP

E7.3 Duality			
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Dual for Diet Problem

Example (Dual of Linear Program for Diet Problem) maximize $\sum_{j=1}^{m} y_j r_j$ subject to

$$\sum_{j=1}^{m} a_{ij} y_j \le c_i \qquad ext{for } 1 \le i \le n$$

 $y_i \ge 0 \qquad ext{for } 1 \le j \le m$

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Linear & Integer ProgrammingDual for Diet Problem: InterpretationExample (Dual of Linear Program for Diet Problem)
maximize
$$\sum_{j=1}^{m} y_j r_j$$
 subject to
 $\sum_{j=1}^{m} a_{ij} y_j \leq c_i$ for $1 \leq i \leq n$
 $y_j \geq 0$ for $1 \leq j \leq m$ Find nutrient prices that maximize total worth of daily nutrients.
The value of nutrients in food F_i may not exceed the cost of F_i .



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