

Planning and Optimization

E3. Landmarks: Introduction & Minimum Hitting Set Heuristic

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Landmarks
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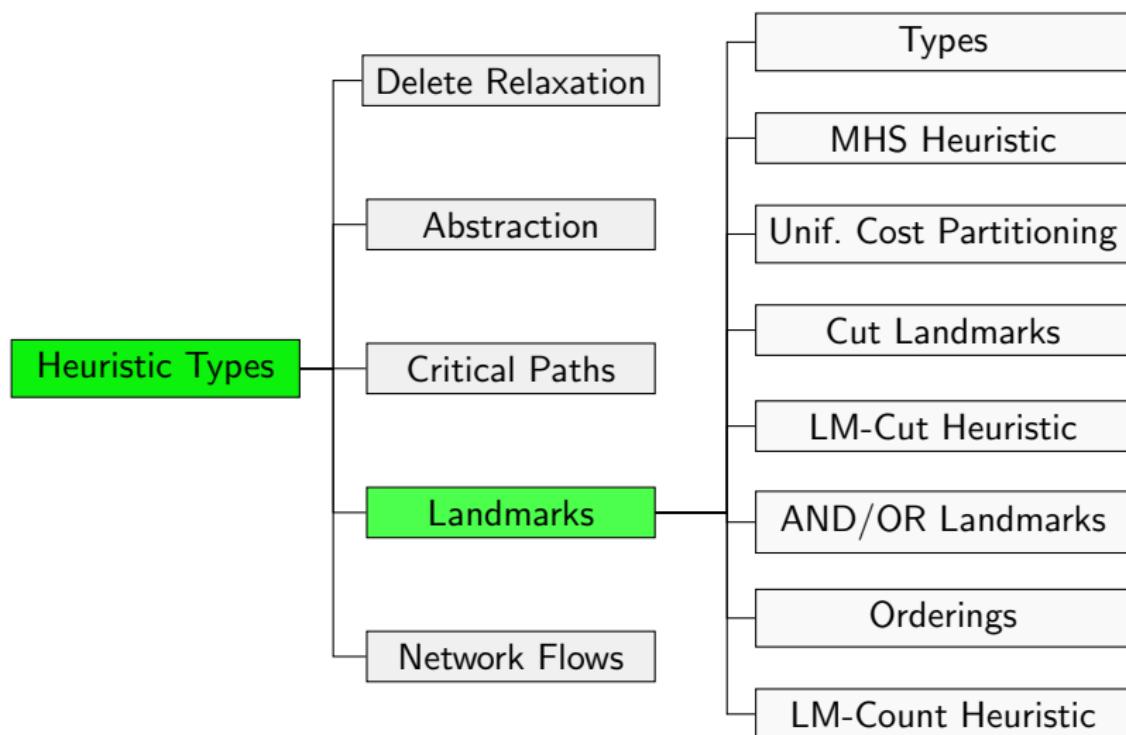
Minimum Hitting Set Heuristic
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Uniform Cost Partitioning
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Summary
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Landmarks

Content of this Course: Heuristic Types



Landmarks

Basic Idea: Something that must happen **in every solution**

For example

- some operator must be applied
- some atom must be true
- some formula must be true

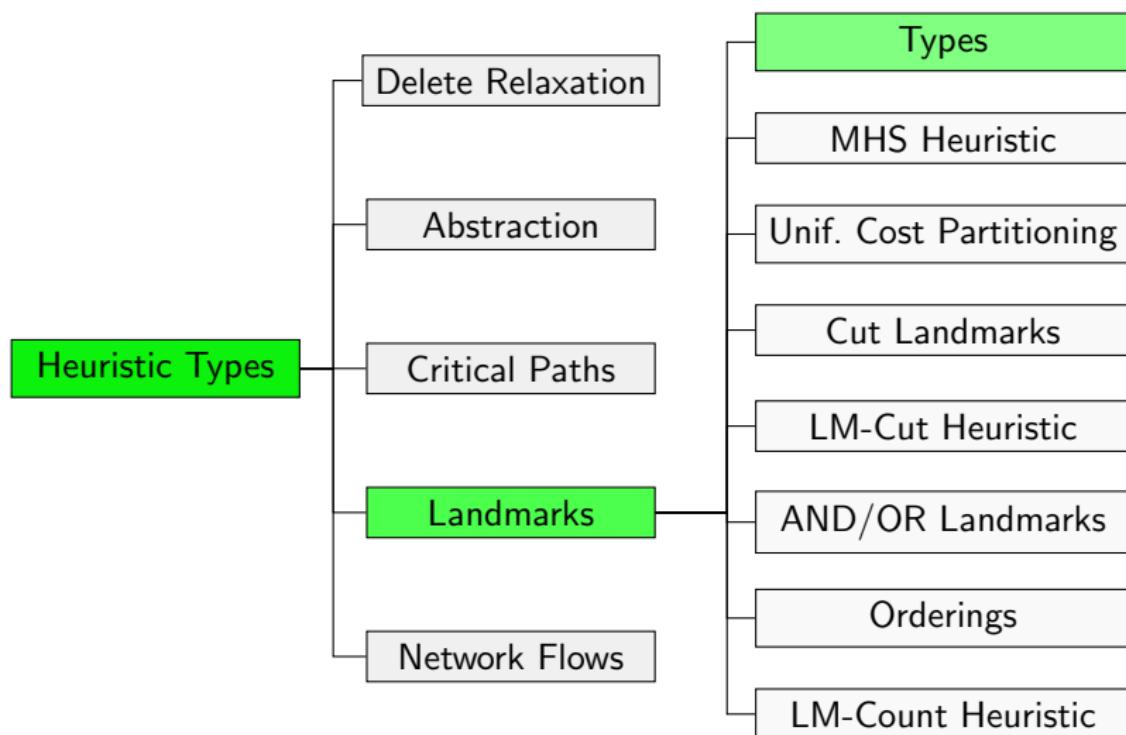
→ Derive heuristic estimate from this kind of information.

Reminder: Terminology

Consider sequence of transitions $s^0 \xrightarrow{\ell_1} s^1, \dots, s^{n-1} \xrightarrow{\ell_n} s^n$
such that $s^0 = s$ and $s^n = s'$.

- s^0, \dots, s^n is called **(state) path** from s to s'
- ℓ_1, \dots, ℓ_n is called **(label) path** from s to s'
- $s^0 \xrightarrow{\ell_1} s^1, \dots, s^{n-1} \xrightarrow{\ell_n} s^n$ is called **trace** from s to s'

Content of this Course: Landmarks



Disjunctive Action Landmarks

Definition (Disjunctive Action Landmark)

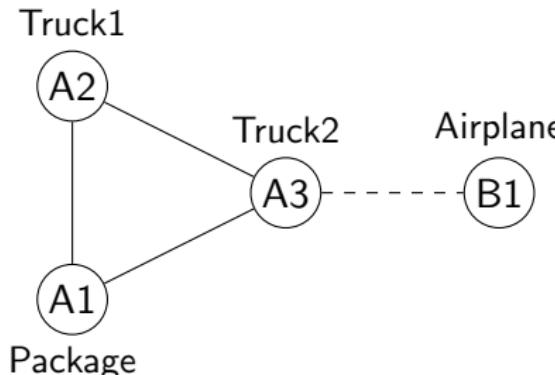
Let s be a state of planning task $\Pi = \langle V, I, O, \gamma \rangle$.

A **disjunctive action landmark** for s is a set of operators $L \subseteq O$ such that every label path from s to a goal state contains an operator from L .

The **cost** of landmark L is $cost(L) = \min_{o \in L} cost(o)$.

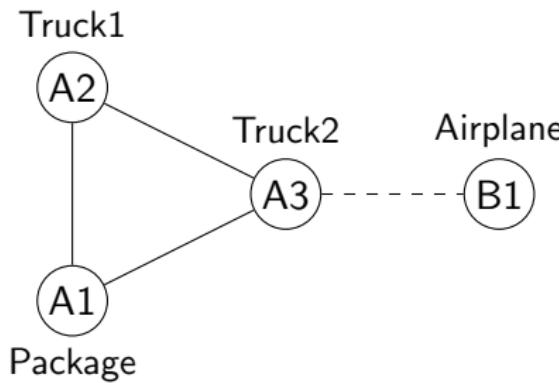
Example Task

- Two trucks, one airplane
- Airplane can fly between locations A3 and B1
- Trucks can drive arbitrarily between locations A1, A2, and A3
- Package to be transported from A1 to B1
- Operators
 - $\text{Load}(v, l)$ and $\text{Unload}(v, l)$ for vehicle v and location l
 - $\text{Drive}(t, l, l')$ for truck t and locations l, l'
 - $\text{Fly}(l, l')$ for locations l, l'



Example: Disjunctive Action Landmarks

$L_1 = \{Load(Truck1, A1), Load(Truck2, A1)\}$ and
 $L_2 = \{Fly(B1, A3)\}$ are disjunctive action landmarks.



What other disjunctive action landmarks are there?

Fact and Formula Landmarks

Definition (Formula and Fact Landmark)

Let s be a state of planning task $\Pi = \langle V, I, O, \gamma \rangle$.

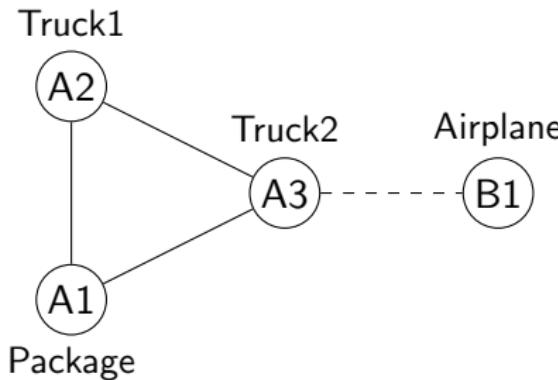
A **formula landmark** for s is a formula λ over V such that every state path from s to a goal state contains a state s' with $s' \models \lambda$.

If $\lambda \in V$ then λ is a **fact landmark**.

Example: Formula Landmarks

$at(Package, A3)$ and $in(Package, Airplane)$ are fact landmarks.

$in(Package, Truck1) \vee in(Package, Truck2)$ is a formula landmark.



What other formula and fact landmarks are there?

Remarks

- Not every landmark is informative. Some examples:
 - If the initial state is not already a goal state then the set of all operators is a disjunctive action landmark.
 - Every variable that is initially true is a fact landmark.
 - The goal formula is a formula landmark.
- Deciding whether a given variable is a fact landmark is as hard as the plan existence problem.
- The same is true for operator sets and disjunctive action landmarks.

Relationship

Disjunctive action landmarks and fact/formula landmarks are related:

- Every fact landmark f that is initially false induces a disjunctive action landmark consisting of all operators that possibly make f true.
- A disjunctive action landmark $\{o_1, \dots, o_n\}$ induces a formula landmark $\lambda = \text{pre}(o_1) \vee \dots \vee \text{pre}(o_n)$ and therefore also a fact landmark v for all $v \in V$ with $\lambda \models v$.

Exploiting Disjunctive Action Landmarks

How can we exploit a given set \mathcal{L} of disjunctive action landmarks?

- Sum of costs $\sum_{L \in \mathcal{L}} \text{cost}(L)$?
~~ not admissible!
- Maximize costs $\max_{L \in \mathcal{L}} \text{cost}(L)$?
~~ usually very weak heuristic
- better: hitting sets or cost partitioning

Landmarks
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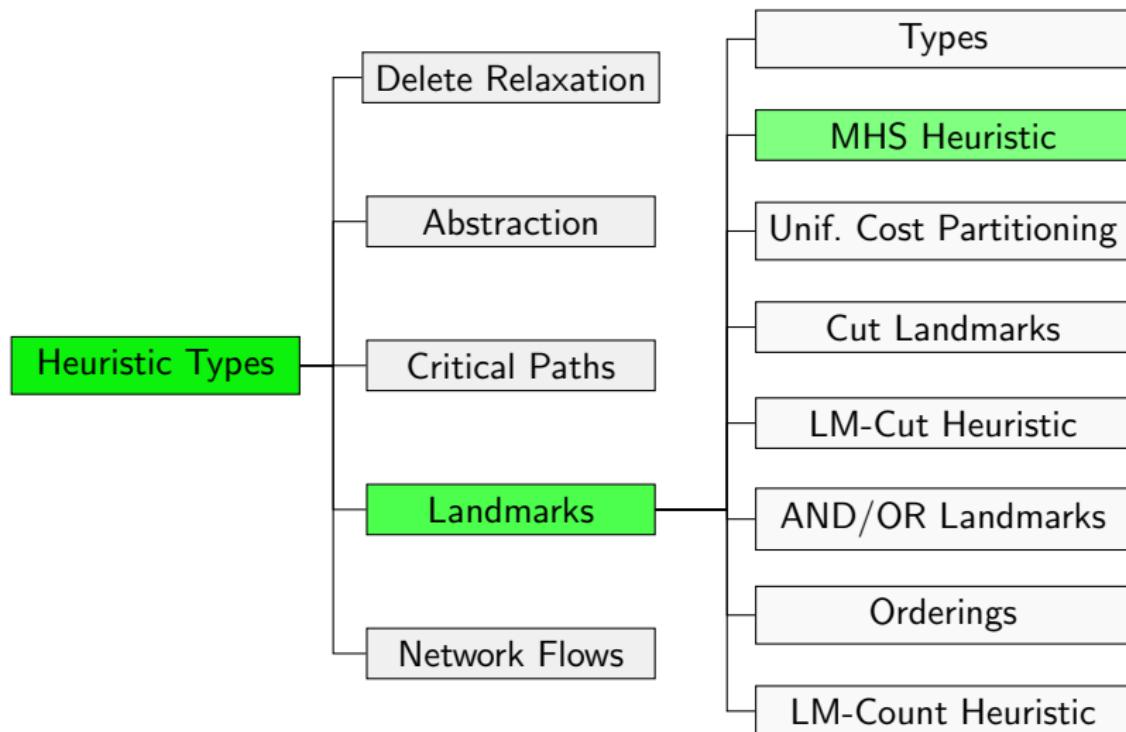
Minimum Hitting Set Heuristic
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Uniform Cost Partitioning
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Minimum Hitting Set Heuristic

Content of this Course: Landmarks



Hitting Sets

Definition (Hitting Set)

Let X be a set, $\mathcal{F} = \{F_1, \dots, F_n\} \subseteq 2^X$ be a family of subsets of X and $c : X \rightarrow \mathbb{R}_0^+$ be a cost function for X .

A **hitting set** is a subset $H \subseteq X$ that “hits” all subsets in \mathcal{F} , i.e., $H \cap F \neq \emptyset$ for all $F \in \mathcal{F}$. The **cost** of H is $\sum_{x \in H} c(x)$.

A **minimum hitting set (MHS)** is a hitting set with minimal cost.

MHS is a “classical” NP-complete problem (Karp, 1972)

Example: Hitting Sets

Example

$$X = \{o_1, o_2, o_3, o_4\}$$

$$\mathcal{F} = \{\{o_4\}, \{o_1, o_2\}, \{o_1, o_3\}, \{o_2, o_3\}\}$$

$$c(o_1) = 3, \quad c(o_2) = 4, \quad c(o_3) = 5, \quad c(o_4) = 0$$

minimum hitting set:

Example: Hitting Sets

Example

$$X = \{o_1, o_2, o_3, o_4\}$$

$$\mathcal{F} = \{\{o_4\}, \{o_1, o_2\}, \{o_1, o_3\}, \{o_2, o_3\}\}$$

$$c(o_1) = 3, \quad c(o_2) = 4, \quad c(o_3) = 5, \quad c(o_4) = 0$$

minimum hitting set: $\{o_1, o_2, o_4\}$ with cost $3 + 4 + 0 = 7$

Hitting Sets for Disjunctive Action Landmarks

Idea: **disjunctive action landmarks** are interpreted as instance of **minimum hitting set**

Definition (Hitting Set Heuristic)

Let \mathcal{L} be a set of disjunctive action landmarks. The **hitting set heuristic** $h^{\text{MHS}}(\mathcal{L})$ is defined as the cost of a minimum hitting set for \mathcal{L} with $c(o) = \text{cost}(o)$.

Proposition (Hitting Set Heuristic is Admissible)

Let \mathcal{L} be a set of disjunctive action landmarks for state s . Then $h^{\text{MHS}}(\mathcal{L})$ is an admissible estimate for s .

Why?

Hitting Set Heuristic: Discussion

- The hitting set heuristic is the **best possible** heuristic that only uses the given information...
- ...but is NP-hard to compute.
- \rightsquigarrow Use approximations that can be efficiently computed.
- Now: [uniform cost partitioning](#)
- Later in the course: [optimal cost partitioning](#)

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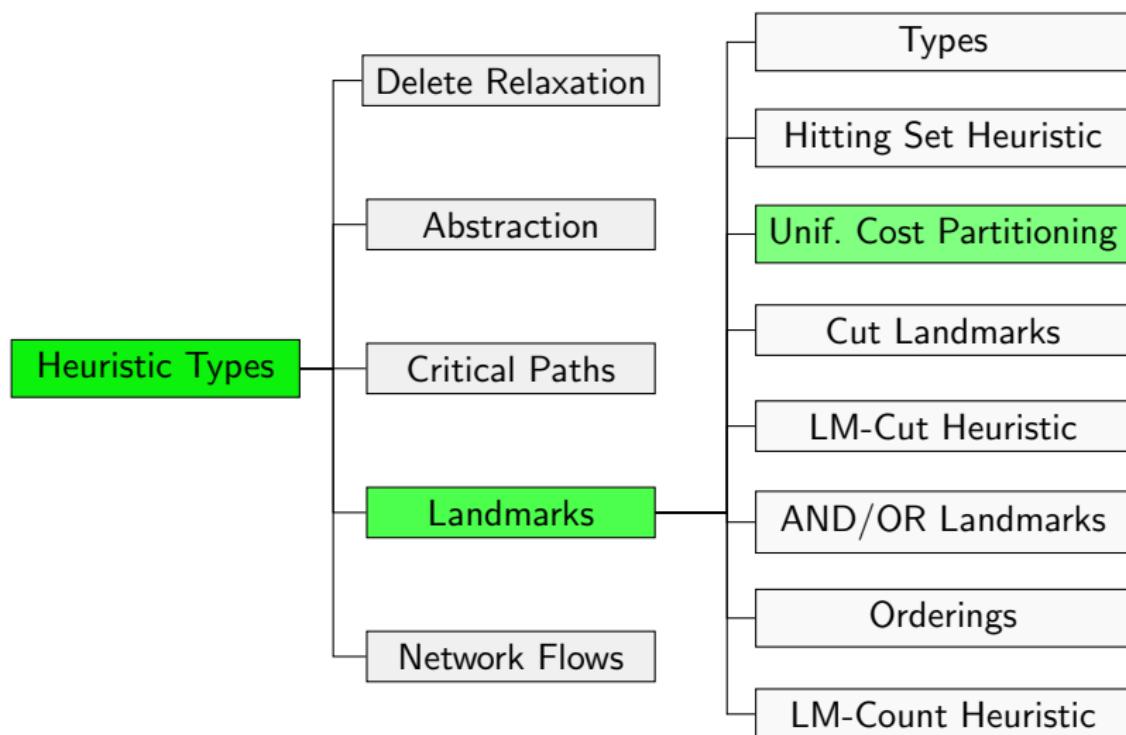
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Uniform Cost Partitioning

Content of this Course: Landmarks



Uniform Cost Partitioning (1)

Idea: Distribute cost of operators uniformly among the landmarks.

Definition (Uniform Cost Partitioning Heuristic for Landmarks)

Let \mathcal{L} be a set of disjunctive action landmarks.

The **uniform cost partitioning heuristic** $h^{\text{UCP}}(\mathcal{L})$ is defined as

$$h^{\text{UCP}}(\mathcal{L}) = \sum_{L \in \mathcal{L}} \min_{o \in L} c'(o) \text{ with}$$

$$c'(o) = \text{cost}(o)/|\{L \in \mathcal{L} \mid o \in L\}|.$$

Uniform Cost Partitioning (2)

Theorem (Uniform Cost Partitioning Heuristic is Admissible)

*Let \mathcal{L} be a set of disjunctive action landmarks for state s of Π .
Then $h^{UCP}(\mathcal{L})$ is an **admissible** heuristic estimate for s .*

Uniform Cost Partitioning (2)

Theorem (Uniform Cost Partitioning Heuristic is Admissible)

Let \mathcal{L} be a set of disjunctive action landmarks for state s of Π . Then $h^{UCP}(\mathcal{L})$ is an **admissible** heuristic estimate for s .

Proof.

Let $\pi = \langle o_1, \dots, o_n \rangle$ be an optimal plan for s . For $L \in \mathcal{L}$ define a new cost function $cost_L$ as $cost_L(o) = c'(o)$ if $o \in L$ and $cost_L(o) = 0$ otherwise. Let Π_L be a modified version of Π , where for all operators o the cost is replaced with $cost_L(o)$. We make three independent observations:

- 1 For $L \in \mathcal{L}$ the value $cost'(L) := \min_{o \in L} c'(o)$ is an admissible estimate for s in Π_L .
- 2 π is also a plan for s in Π_L , so $h_{\Pi_L}^*(s) \leq \sum_{i=1}^n cost_L(o_i)$.
- 3 $\sum_{L \in \mathcal{L}} cost_L(o) = cost(o)$ for each operator o .

Uniform Cost Partitioning (3)

Proof (continued).

Together, this leads to the following inequality (subscripts indicate for which task the heuristic is computed):

$$\begin{aligned}
 h_{\Pi}^{\text{UCP}}(\mathcal{L}) &= \sum_{L \in \mathcal{L}} \text{cost}'(L) \stackrel{(1)}{\leq} \sum_{L \in \mathcal{L}} h_{\Pi_L}^*(s) \\
 &\stackrel{(2)}{\leq} \sum_{L \in \mathcal{L}} \sum_{i=1}^n \text{cost}_L(o_i) = \sum_{i=1}^n \sum_{L \in \mathcal{L}} \text{cost}_L(o_i) \\
 &\stackrel{(3)}{=} \sum_{i=1}^n \text{cost}(o) = h_{\Pi}^*(s)
 \end{aligned}$$



Relationship

Theorem

Let \mathcal{L} be a set of disjunctive action landmarks for state s .

Then $h^{UCP}(\mathcal{L}) \leq h^{MHS}(\mathcal{L}) \leq h^*(s)$.

(Proof omitted.)

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Summary

- **Landmarks** describe properties that are shared by all plans of a task.
- **Hitting sets** yield the most accurate heuristic for a given set of disjunctive action landmarks, but the computation is NP-hard.
- **Uniform cost partitioning** is a polynomial approach for the computation of informative heuristics from disjunctive action landmarks.