Planning and Optimization

E2. Critical Path Heuristics: Properties and Π^m Compilation

Malte Helmert and Gabriele Röger

Universität Basel

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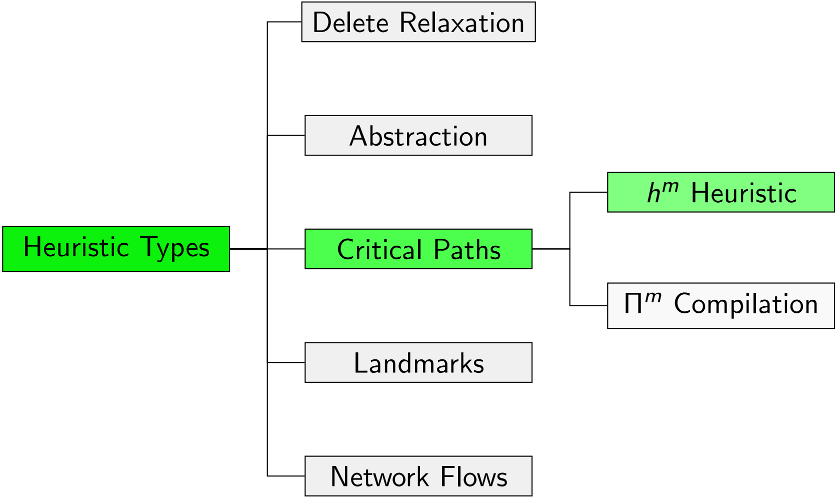
Critical Path Heuristics



Heuristic Properties

Heuristic for Forward or Backward Search? (1)

Any heuristic can be used for both, forward and backward search:

- Let h_f be a forward search heuristic (as in earlier chapters). We can use it to get estimate for state S in backward search on task (V, I, O, G), computing $h_f(I)$ on task (V, I, O, S).

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- We also can use a backward search heuristic h_b in forward search on task (V, I, O, G), determining estimate for state s as $h_b(G)$ on task (V, s, O, G).

Heuristic for Forward or Backward Search? (2)

We defined h^m so that it can directly be used for both directions on task (V, I, O, G) as

- $h^m_f(s) := h^m(s, G)$ for forward search, or
- $h^m_b(S) := h^m(I, S)$ for backward search.

Precomputation determines $h^m(s, B)$ for all $B \subseteq V$ with $|B| \leq m$.

- For h^m_f, we can only use these values for a single heuristic evaluation, because the state s changes.
- For h^m_b, we can re-use these values and all subsequent heuristic evaluations are quite cheap.

$\rightarrow$ h^m better suited for backward search

$\rightarrow$ We examine it in the following in this context.

Heuristic Properties (1)

Theorem

Let $\Pi = \langle V, I, O, G \rangle$ be a conflict-free STRIPS planning tasks and $S \subseteq V$ be a backward search state. Then $h_b^m(S) := h^m(I, S)$ is a safe, goal-aware, consistent, and admissible heuristic for Π.

Proof.

We prove goal-awareness and consistency, the other properties follow from these two.

Goal-awareness: S is a goal state iff $S \subseteq I$. Then $h_b^m(S) = h^m(I, S) = 0$. ...

Heuristic Properties (2)

Proof (continued).

Consistency: Assume h_b^m is not consistent, i.e., there is a state S and an operator o, where $R := sregr(S, o) \neq \bot$ such that $h_b^m(S) > cost(o) + h_b^m(R)$.

Then $h_b^m(S) = h^m(I, S)$ and there is $S' \subseteq S$ with $|S'| \leq m$ and $h^m(I, S') = h^m(I, S)$: if $|S| \leq m$, choose $S' = S$, otherwise choose any maximizing subset from the last h^m equation.

As $S' \subseteq S$ and $sregr(S, o) \neq \bot$, also $R' := sregr(S', o) \neq \bot$ and $(R', o) \in R(S', O)$. This gives $h^m(I, S') \leq cost(o) + h^m(I, R')$.

As $S' \subseteq S$, it holds that $R' \subseteq R$ and $h^m(I, R') \leq h^m(I, R)$.

Overall, we get $h_b^m(S) = h^m(I, S) = h^m(I, S') \leq cost(o) + h^m(I, R') \leq cost(o) + h^m(I, R) = cost(o) + h_b^m(R)$. ↯ □

Heuristic Properties (3)

Theorem

For $m, m' \in \mathbb{N}_1$ with $m < m'$ it holds that $h^m \leq h^{m'}$.

(Proof omitted.)

Heuristic Properties (4)

Theorem

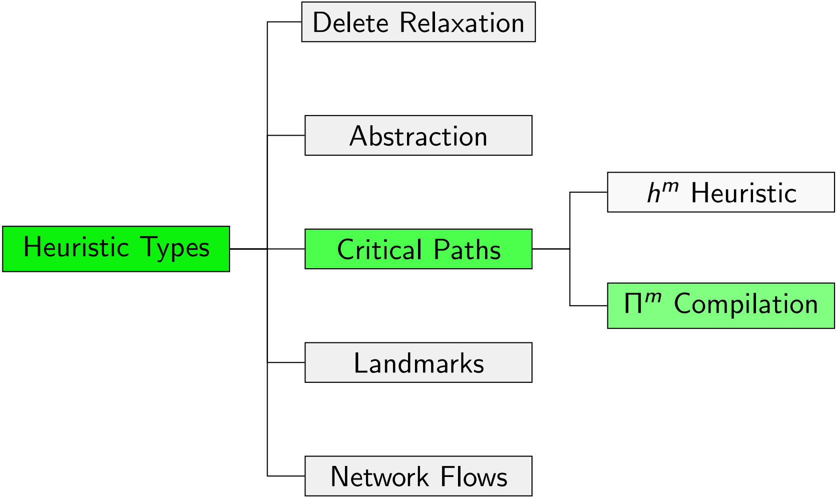
Let $\Pi = \langle V, I, O, G \rangle$ be a conflict-free STRIPS planning task. For a sufficiently large m, it holds that $h^m = r^*$ on Π.

Proof Sketch.

It is easy to check that for $m = |V|$ the heuristic definition of h^m can be simplified so that it becomes the definition of r^*.

Π^m Compilation

Critical Path Heuristics



Π^m Compilation: Motivation

- We have seen that $h^1 = h^{\max}$ and that $h^{\max}$ corresponds to the cost of a critical path in the relaxed task graph.
- What about $m > 1$?
- Π^m compilation derives for a given m a task Π^m from the original task Π.
- h^m corresponds to cost of critical path in the relaxed task graph of Π^m.

$\rightarrow$ Better understanding of h^m

$\rightarrow$ Also interesting in the context of landmark heuristics

Idea of Π^m Compilation

- $h^{\max}$ only considers variables individually.
- For example, it cannot detect that a goal $\{a, b\}$ is unreachable from the empty set if every action that adds a deletes b and vice versa.
- Idea: Use meta-variable $v_{\{a,b\}}$ to capture such interactions.
- Intuitively $v_{\{a,b\}}$ is reachable in Π^m if a state where a and b are both true would be reachable in Π when only capturing interactions of at most m variables.

Some Notation

- For a set X of variables and $m \in \mathbb{N}_1$ we define $X^m := \{v_Y \mid Y \subseteq X, |Y| \leq m\}$.
- Example: $\{a, b, c\}^2 = \{v_\emptyset, v_{\{a\}}, v_{\{b\}}, v_{\{c\}}, v_{\{a,b\}}, v_{\{a,c\}}, v_{\{b,c\}}\}$

Π^m Compilation

Definition (Π^m)

Let $\Pi = \langle V, I, O, G \rangle$ be a conflict-free STRIPS planning task. For $m \in \mathbb{N}_1$, the task Π^m is the STRIPS planning task $\langle V^m, I^m, O^m, G^m \rangle$, where

$O^m = \{a_{o,S} \mid o \in O, S \subseteq V, |S| < m, S \cap (add(o) \cup del(o)) = \emptyset\}$

with

- $pre(a_{o,S}) = (pre(o) \cup S)^m$
- $add(a_{o,S}) = \{v_Y \mid Y \subseteq add(o) \cup S, |Y| \leq m, Y \cap add(o) \neq \emptyset\}$
- $del(a_{o,S}) = \emptyset$
- $cost(a_{o,S}) = cost(o)$

Π^m for Running Example with $m = 2$

For running example Π we get $\Pi^2 = \langle V', I', O', G' \rangle$, where

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$V = \{a, b, c\}$
$V' = V^2 = \{v_Y \mid Y \subseteq V, |Y| \leq 2\}$

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$$V' = \{v_\emptyset, v_{\{a\}}, v_{\{b\}}, v_{\{c\}}, v_{\{a,b\}}, v_{\{a,c\}}, v_{\{b,c\}}\}$$
$$I' = \{v_\emptyset, v_{\{a\}}\}$$

$$I = \{a\}$$
$$I' = I^2 = \{v_Y \mid Y \subseteq I, |Y| \leq 2\}$$

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$$I' = \{v_\emptyset, v_{\{a\}}\}$$
$$G' = \{v_\emptyset, v_{\{a\}}, v_{\{b\}}, v_{\{c\}}, v_{\{a,b\}}, v_{\{a,c\}}, v_{\{b,c\}}\}$$

$G = \{a, b, c\}$
$G' = G^2 = \{v_Y \mid Y \subseteq G, |Y| \leq 2\}$

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$$O' = \{a_{o_1,\emptyset}, a_{o_1,\{a\}}, a_{o_2,\emptyset}, a_{o_2,\{c\}}, a_{o_3,\emptyset}, a_{o_3,\{b\}}, a_{o_3,\{c\}}\}$$

$$o_1 = \langle \{a, b\}, \{c\}, \{b\}, 1 \rangle$$
$$o_2 = \langle \{a\}, \{b\}, \{a\}, 2 \rangle$$
$$o_3 = \langle \{b\}, \{a\}, \emptyset, 2 \rangle$$
$$O' = \{a_{o,S} \mid o \in O, S \subseteq V, |S| < m, S \cap (add(o) \cup del(o)) = \emptyset\}$$

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\end{aligned}
$$

with (for example)

$$a_{o_3,\{c\}} = \langle \{v_\emptyset, v_{\{b\}}, v_{\{c\}}, v_{\{b,c\}}\}, \ldots, \ldots, \ldots \rangle$$

$o_3 = \langle \{b\}, \{a\}, \emptyset, 2 \rangle$
$pre(a_{o,S}) = (pre(o) \cup S)^2$

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$o_3 = \langle \{b\}, \{a\}, \emptyset, 2 \rangle$

$add(a_{o,S}) = \{v_Y \mid Y \subseteq add(o) \cup S, |Y| \leq m, Y \cap add(o) \neq \emptyset\}$

Π^m for Running Example with $m = 2$

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$o_3 = \langle\{b\}, \{a\}, \emptyset, 2\rangle$
$del(a_{o,s}) = \emptyset$

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$o_3 = \langle \{b\}, \{a\}, \emptyset, 2 \rangle$
$cost(a_{o,S}) = cost(o)$

Π^m: Properties

Theorem ($h^m_\Pi = h^{\max}_{\Pi^m}$)

Let Π be a conflict-free STRIPS planning task and $m \in \mathbb{N}_1$.

Then for each state s of Π it holds that $h^m_\Pi(s) = h^{max}_{\Pi^m}(s^m)$, where the subscript denotes on which task the heuristic is computed.

(Proof omitted.)

Can we in general compute an admissible heuristic on Π^m and get admissible estimates for Π? $\rightsquigarrow$ No!

Theorem

There are conflict-free STRIPS planning tasks Π, $m \in \mathbb{N}_1$ *and admissible heuristics* h *such that* $h^*_\Pi(s) < h_{\Pi^m}(s^m)$ *for some state* s *of* Π.

(Proof omitted.)

Summary

Summary

- h^m heuristics are best suited for backward search.
- h^m heuristics are safe, goal aware, consistent and admissible.
- The Π^m compilation explicitly represents sets ($\hat{=}$ conjunctions) of variables as meta-variables.
- $h^m_\Pi(s) = h^{\max}_{\Pi^m}(s^m)$

Literature

Literature

References on critical path heuristics:

Patrik Haslum and Hector Geffner.
Admissible Heuristics for Optimal Planning.
Proc. AIPS 2000, pp. 140–149, 2000.
Introduces h^m heuristics.

Patrik Haslum.
$h^m(P) = h^1(P^m)$: Alternative Characterisations of the Generalisation From $h^{\max}$ to h^m.
Proc. ICAPS 2009, pp. 354–357, 2009.
Introduces Π^m compilation.