

Planning and Optimization

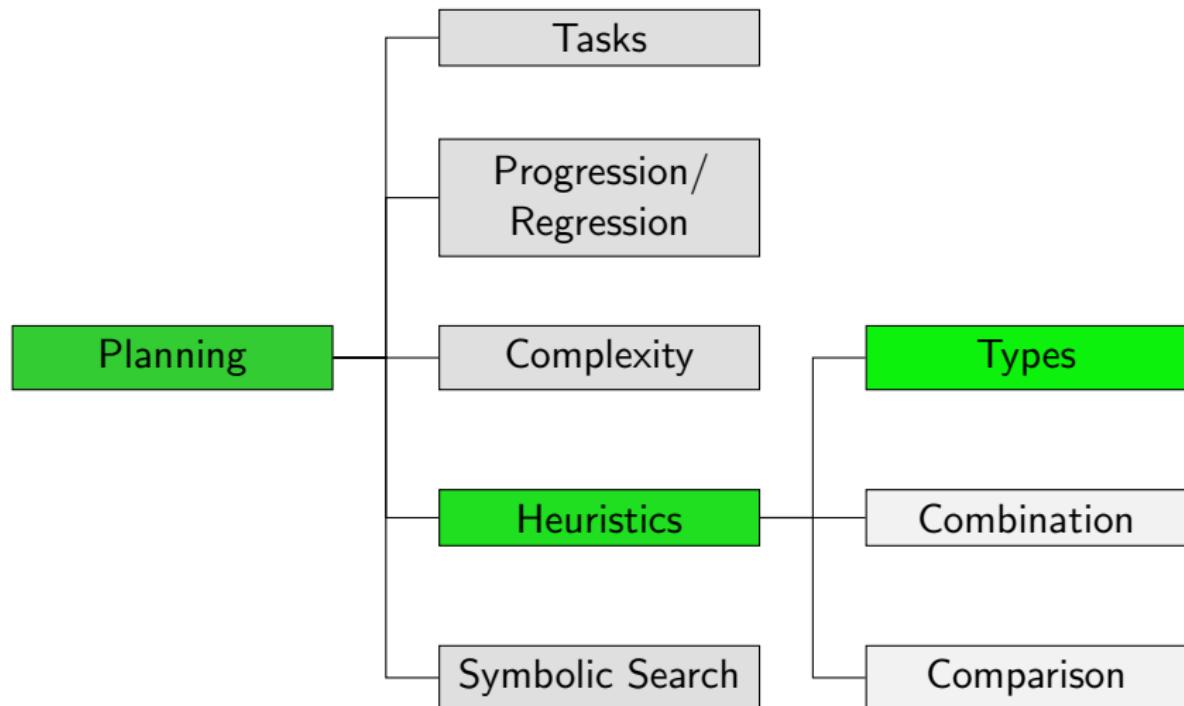
E1. Critical Path Heuristics: h^m

Malte Helmert and Gabriele Röger

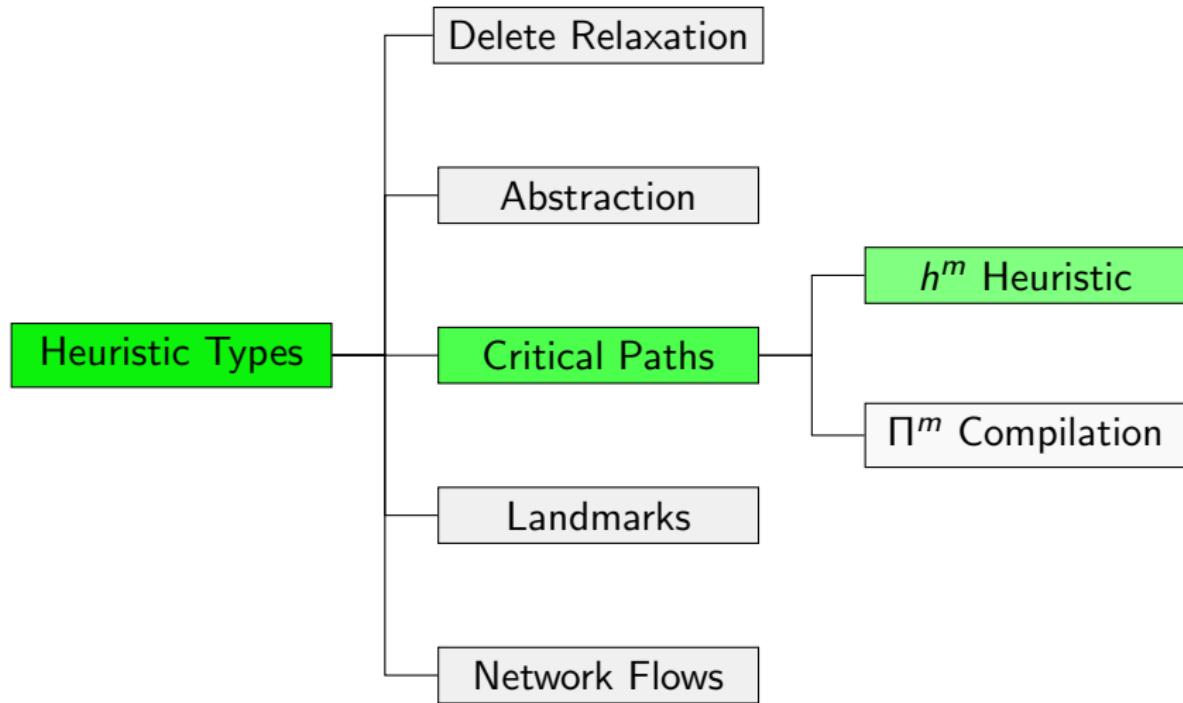
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Content of this Course



Content of this Course: Heuristic Types



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Set Representation

In This (and the Next) Chapter...

- ...we consider only STRIPS, and ...
- ...we focus on backward search and regression.

Set Representation of STRIPS Planning Tasks

For a more convenient notation, we will use a set representation of STRIPS planning task...

Three differences:

- Represent conjunctions of variables as sets of variables.
- Use two sets to represent add and delete effects of operators separately.
- Represent states as sets of the true variables.

STRIPS Operators in Set Representation: Reminder

- Every STRIPS operator is of the form

$$\langle v_1 \wedge \cdots \wedge v_p, \quad a_1 \wedge \cdots \wedge a_q \wedge \neg d_1 \wedge \cdots \wedge \neg d_r, c \rangle$$

where v_i, a_j, d_k are state variables and c is the cost.

- The same operator o in **set representation** is $\langle \text{pre}(o), \text{add}(o), \text{del}(o), \text{cost}(o) \rangle$, where
 - $\text{pre}(o) = \{v_1, \dots, v_p\}$ are the **preconditions**,
 - $\text{add}(o) = \{a_1, \dots, a_q\}$ are the **add effects**,
 - $\text{del}(o) = \{d_1, \dots, d_r\}$ are the **delete effects**, and
 - $\text{cost}(o) = c$ is the operator cost.

STRIPS Planning Tasks in Set Representation

A **STRIPS planning task in set representation** is given as a tuple $\langle V, I, O, G \rangle$, where

- V is a finite set of state variables,
- $I \subseteq V$ is the initial state,
- O is a finite set of STRIPS operators in set representation,
- $G \subseteq V$ is the goal.

STRIPS Planning Tasks in Set Representation

A **STRIPS planning task in set representation** is given as a tuple $\langle V, I, O, G \rangle$, where

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- $I \subseteq V$ is the initial state,
- O is a finite set of STRIPS operators in set representation,
- $G \subseteq V$ is the goal.

The corresponding planning task in the previous notation is $\langle V, I', O', \gamma \rangle$, where

- $I'(v) = \top$ iff $v \in I$,
- $O' = \{ \langle \bigwedge_{v \in \text{pre}(o)} v, \bigwedge_{v \in \text{add}(o)} v \wedge \bigwedge_{v \in \text{del}(o)} \neg v, \text{cost}(o) \rangle \mid o \in O \}$,
- $\gamma = \bigwedge_{v \in G} v$.

Reminder: STRIPS Regression

Definition (STRIPS Regression)

Let $\varphi = \varphi_1 \wedge \dots \wedge \varphi_n$ be a conjunction of atoms, and let o be a conflict-free STRIPS operator which adds the atoms a_1, \dots, a_k and deletes the atoms d_1, \dots, d_l . (W.l.o.g., $a_i \neq d_j$ for all i, j .)

The **STRIPS regression** of φ with respect to o is

$$sregr(\varphi, o) := \begin{cases} \perp & \text{if } \varphi_i = d_j \text{ for some } i, j \\ pre(o) \wedge \bigwedge (\{\varphi_1, \dots, \varphi_n\} \setminus \{a_1, \dots, a_k\}) & \text{else} \end{cases}$$

Note: $sregr(\varphi, o)$ is again a conjunction of atoms, or \perp .

STRIPS Regression in Set Representation

Definition (STRIPS Regression)

Let A be a set of atoms, and let o be a conflict-free STRIPS operator $o = \langle \text{pre}(o), \text{add}(o), \text{del}(o), \text{cost}(o) \rangle$.
(W.l.o.g., $\text{add}(o) \cap \text{del}(o) = \emptyset$.)

The **STRIPS regression** of A with respect to o is

$$\text{sregr}(A, o) := \begin{cases} \perp & \text{if } A \cap \text{del}(o) \neq \emptyset \\ \text{pre}(o) \cup (A \setminus \text{add}(o)) & \text{otherwise} \end{cases}$$

Note: $\text{sregr}(A, o)$ is again a set of atoms, or \perp .

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Perfect Regression Heuristic

Perfect Regression Heuristic

Definition (Perfect Regression Heuristic)

For a conflict-free STRIPS planning task $\langle V, I, O, G \rangle$ the **perfect regression heuristic r^*** for state s and variable set $A \subseteq V$ is defined as the (point-wise) greatest fixed-point solution of the equations:

$$r^*(s, A) = \begin{cases} 0 & \text{if } A \subseteq s \\ \min_{(B, o) \in R(A, O)} [cost(o) + r^*(s, B)] & \text{otherwise} \end{cases}$$

$$R(A, O) = \{(B, o) \mid o \in O, B = sregr(A, o) \neq \perp\}$$

Perfect Regression Heuristic r^* vs. Perfect Heuristic h^*

Theorem

For a conflict-free STRIPS planning task $\langle V, I, O, G \rangle$ it holds for each state s that $h^*(s) = r^*(s, G)$.

Intuition: We can extract a path from the operators in the minimizing pairs (B, o) , starting from the goal.

~ r^* cannot be computed efficiently.

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Critical Path Heuristics

Running Example

We will use the following running example throughout this chapter:

$\Pi = \langle V, I, \{o_1, o_2, o_3\}, G \rangle$ with

$$V = \{a, b, c\}$$

$$I = \{a\}$$

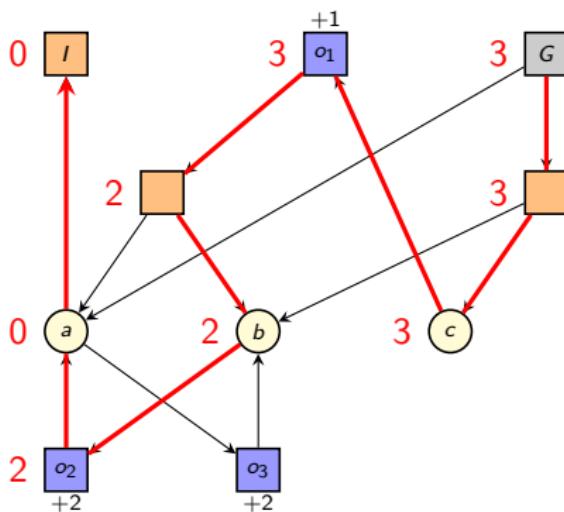
$$o_1 = \langle \{a, b\}, \{c\}, \{b\}, 1 \rangle$$

$$o_2 = \langle \{a\}, \{b\}, \{a\}, 2 \rangle$$

$$o_3 = \langle \{b\}, \{a\}, \emptyset, 2 \rangle$$

$$G = \{a, b, c\}$$

Optimal plan o_2, o_3, o_1, o_2, o_3 has cost 9.

Back to RTG and h^{\max} 

The critical path justifies the heuristic estimate $h^{\max}(I) = 3$

h^{\max} as Critical Path Heuristic

Definition (h^{\max} Heuristic)

For a conflict-free STRIPS planning task $\langle V, I, O, G \rangle$ the heuristic h^{\max} for state s and variable set $A \subseteq V$ is defined as the (point-wise) greatest fixed-point solution of

$$h^{\max}(s, A) =$$

$$\begin{cases} 0 & \text{if } A \subseteq s \\ \min_{(B, o) \in R(A, O)} [\text{cost}(o) + h^{\max}(s, B)] & \text{if } |A| \leq 1 \text{ and } A \not\subseteq s \\ \max_{v \in A} h^{\max}(s, \{v\}) & \text{otherwise} \end{cases}$$

$$R(A, O) = \{\langle B, o \rangle \mid o \in O, B = \text{sreg}(A, o) \neq \perp\}$$

Estimate $r^*(s, A)$ as cost of most expensive $v \in A$.

This definition specifies the same heuristic h^{\max} as in the chapter on relaxation heuristics.

Critical Path Heuristics

Definition (h^m Heuristics)

For a conflict-free STRIPS planning task $\langle V, I, O, G \rangle$ and $m \in \mathbb{N}_1$ the heuristic h^m for state s and variable set $A \subseteq V$ is defined as the (point-wise) greatest fixed-point solution of

$$h^m(s, A) =$$

$$\begin{cases} 0 & \text{if } A \subseteq s \\ \min_{\langle B, o \rangle \in R(A, O)} [cost(o) + h^m(s, B)] & \text{if } |A| \leq m \text{ and } A \not\subseteq s \\ \max_{B \subseteq A, 1 \leq |B| \leq m} h^m(s, B) & \text{otherwise} \end{cases}$$

$$R(A, O) = \{\langle B, o \rangle \mid o \in O, B = sregr(A, o) \neq \perp\}$$

Estimate $r^*(s, A)$ as cost of most expensive $B \subseteq A$ with $|B| \leq m$.

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Critical Path Heuristics: Computation

Definition (h^m Heuristics)

For a conflict-free STRIPS planning task $\langle V, I, O, G \rangle$ and $m \in \mathbb{N}_1$ the heuristic h^m for state s and variable set $A \subseteq V$ is defined as the (point-wise) greatest fixed-point solution of

$$h^m(s, A) =$$

$$\begin{cases} 0 & \text{if } A \subseteq s \\ \min_{(B,o) \in R(A,O)} [\text{cost}(o) + h^m(s, B)] & \text{if } |A| \leq m \text{ and } A \not\subseteq s \\ \max_{B \subseteq A, 1 \leq |B| \leq m} h^m(s, B) & \text{otherwise} \end{cases}$$

$$R(A, O) = \{\langle B, o \rangle \mid o \in O, B = \text{sregr}(A, o) \neq \perp\}$$

Cheap to evaluate given $h^m(s, B)$ for all $B \subseteq V$ with $1 \leq |B| \leq m$.
We precompute these values.

h^m Precomputation (1)

For value m and state s of task with variables V and operators O

Computing h^m Values for Variable Sets up to Size m

$S := \{A \subseteq V \mid |A| \leq m\}$

Associate a *cost* attribute with each set $A \in S$.

for all sets $A \in S$:

if $A \subseteq s$ **then** $A.cost := 0$

else $A.cost := \infty$

while no fixed point is reached:

Choose a variable set A from S .

$newcost := \min_{\langle B, o \rangle \in R(A, O)} [cost(o) + currentcost(B, S)]$

if $newcost < A.cost$ **then** $A.cost := newcost$

currentcost(B, S)

if $|B| \leq m$ **then return** $B.cost$ **else return** $\max_{A \in S, A \subseteq B} A.cost$

h^m Precomputation (2)

- Fixed point reached $\Rightarrow A.\text{cost} = h^m(s, A)$ for all $A \in S$.
- Intuition:
 - cost values satisfy h^m equations, and
 - no larger values can satisfy the equations: initialized to ∞ and values are only reduced if it is otherwise impossible to satisfy an equation.

h^m Precomputation (2)

- Fixed point reached $\Rightarrow A.\text{cost} = h^m(s, A)$ for all $A \in S$.
- Intuition:
 - cost values satisfy h^m equations, and
 - no larger values can satisfy the equations: initialized to ∞ and values are only reduced if it is otherwise impossible to satisfy an equation.
- With suitable data structures, we can choose A in each iteration so that it directly gets assigned its final value (Generalized Dijkstra's algorithm).
- With such a strategy, the runtime is **polynomial for fixed m** .
- Runtime is **exponential in m** $\rightsquigarrow h^m$ typically used with $m \leq 3$

Example with $m = 1$ to Initial State

$$R(\{a\}, \{o_1, o_2, o_3\}) = \{(\{a, b, c\}, o_1), (\{b\}, o_3)\}$$

$$R(\{b\}, \{o_1, o_2, o_3\}) = \{(\{a\}, o_2), (\{b\}, o_3)\}$$

$$R(\{c\}, \{o_1, o_2, o_3\}) = \{(\{a, b\}, o_1), (\{a, c\}, o_2), (\{b, c\}, o_3)\}$$

Example with $m = 1$ to Initial State

$$R(\{a\}, \{o_1, o_2, o_3\}) = \{(\{a, b, c\}, o_1), (\{b\}, o_3)\}$$

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$$R(\{c\}, \{o_1, o_2, o_3\}) = \{(\{a, b\}, o_1), (\{a, c\}, o_2), (\{b, c\}, o_3)\}$$

	$\{a\}$	$\{b\}$	$\{c\}$
$cost$	0	∞	∞

$$\{b\}: \min\{2 + \{a\}.cost, 2 + \{b\}.cost\} = 2$$

Example with $m = 1$ to Initial State

$$R(\{a\}, \{o_1, o_2, o_3\}) = \{(\{a, b, c\}, o_1), (\{b\}, o_3)\}$$

$$R(\{b\}, \{o_1, o_2, o_3\}) = \{(\{a\}, o_2), (\{b\}, o_3)\}$$

$$R(\{c\}, \{o_1, o_2, o_3\}) = \{(\{a, b\}, o_1), (\{a, c\}, o_2), (\{b, c\}, o_3)\}$$

	$\{a\}$	$\{b\}$	$\{c\}$
$cost$	0	2	∞

$$\{b\}: \min\{2 + \{a\}.cost, 2 + \{b\}.cost\} = 2$$

$$\begin{aligned}\{c\}: \min\{ &1 + \max\{\{a\}.cost, \{b\}.cost\}, \\ &2 + \max\{\{a\}.cost, \{c\}.cost\}, \\ &2 + \max\{\{b\}.cost, \{c\}.cost\} \} = 3\end{aligned}$$

Example with $m = 1$ to Initial State

$$R(\{a\}, \{o_1, o_2, o_3\}) = \{(\{a, b, c\}, o_1), (\{b\}, o_3)\}$$

$$R(\{b\}, \{o_1, o_2, o_3\}) = \{(\{a\}, o_2), (\{b\}, o_3)\}$$

$$R(\{c\}, \{o_1, o_2, o_3\}) = \{(\{a, b\}, o_1), (\{a, c\}, o_2), (\{b, c\}, o_3)\}$$

	$\{a\}$	$\{b\}$	$\{c\}$
$cost$	0	2	3

$$\{b\}: \min\{2 + \{a\}.cost, 2 + \{b\}.cost\} = 2$$

$$\begin{aligned}\{c\}: \min\{ &1 + \max\{\{a\}.cost, \{b\}.cost\}, \\ &2 + \max\{\{a\}.cost, \{c\}.cost\}, \\ &2 + \max\{\{b\}.cost, \{c\}.cost\} \} = 3\end{aligned}$$

Example with $m = 1$ to Initial State

	$\{a\}$	$\{b\}$	$\{c\}$
$cost$	0	2	3

$$\{b\}: \min\{2 + \{a\}.cost, 2 + \{b\}.cost\} = 2$$

$$\begin{aligned}\{c\}: \min\{1 + \max\{\{a\}.cost, \{b\}.cost\}, \\ 2 + \max\{\{a\}.cost, \{c\}.cost\}, \\ 2 + \max\{\{b\}.cost, \{c\}.cost\}\} = 3\end{aligned}$$

Fixed point reached

Example with $m = 1$ to Initial State

	$\{a\}$	$\{b\}$	$\{c\}$
$cost$	0	2	3

$$\{b\}: \min\{2 + \{a\}.cost, 2 + \{b\}.cost\} = 2$$

$$\begin{aligned}\{c\}: \min\{1 + \max\{\{a\}.cost, \{b\}.cost\}, \\ 2 + \max\{\{a\}.cost, \{c\}.cost\}, \\ 2 + \max\{\{b\}.cost, \{c\}.cost\}\} = 3\end{aligned}$$

Fixed point reached

$$\begin{aligned}h^1(I, \{a, b, c\}) &= \max\{h^1(I, \{a\}), h^1(I, \{b\}), h^1(I, \{c\})\} \\ &= \max\{0, 2, 3\} = 3\end{aligned}$$

Example with $m = 2$ to Initial State

	$\{a\}$	$\{b\}$	$\{c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
$cost$	0	∞	∞	∞	∞	∞

$$\{b\}: \min\{2 + \{a\}.cost, 2 + \{b\}.cost\} = 2$$

Example with $m = 2$ to Initial State

	$\{a\}$	$\{b\}$	$\{c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
$cost$	0	2	∞	∞	∞	∞

$$\begin{aligned}\{b\}: \quad & \min\{2 + \{a\}.cost, 2 + \{b\}.cost\} = 2 \\ \{a, b\}: \quad & \min\{2 + \{b\}.cost\} = 4\end{aligned}$$

Example with $m = 2$ to Initial State

	$\{a\}$	$\{b\}$	$\{c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
$cost$	0	2	∞	4	∞	∞

$$\{b\}: \min\{2 + \{a\}.cost, 2 + \{b\}.cost\} = 2$$

$$\{a, b\}: \min\{2 + \{b\}.cost\} = 4$$

$$\{c\}: \min\{1 + \{a, b\}.cost, 2 + \{a, c\}.cost, 2 + \{b, c\}.cost\} = 5$$

Example with $m = 2$ to Initial State

	$\{a\}$	$\{b\}$	$\{c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
$cost$	0	2	5	4	∞	∞

$$\{b\}: \min\{2 + \{a\}.cost, 2 + \{b\}.cost\} = 2$$

$$\{a, b\}: \min\{2 + \{b\}.cost\} = 4$$

$$\{c\}: \min\{1 + \{a, b\}.cost, 2 + \{a, c\}.cost, 2 + \{b, c\}.cost\} = 5$$

$$\{a, c\}: \min\{1 + \{a, b\}.cost, 2 + \{b, c\}.cost\} = 5$$

Example with $m = 2$ to Initial State

	$\{a\}$	$\{b\}$	$\{c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
$cost$	0	2	5	4	5	∞

$$\{b\}: \min\{2 + \{a\}.cost, 2 + \{b\}.cost\} = 2$$

$$\{a, b\}: \min\{2 + \{b\}.cost\} = 4$$

$$\{c\}: \min\{1 + \{a, b\}.cost, 2 + \{a, c\}.cost, 2 + \{b, c\}.cost\} = 5$$

$$\{a, c\}: \min\{1 + \{a, b\}.cost, 2 + \{b, c\}.cost\} = 5$$

$$\{b, c\}: \min\{2 + \{a, c\}.cost, 2 + \{b, c\}.cost\} = 7$$

Example with $m = 2$ to Initial State

	$\{a\}$	$\{b\}$	$\{c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
$cost$	0	2	5	4	5	7

$$\{b\}: \min\{2 + \{a\}.cost, 2 + \{b\}.cost\} = 2$$

$$\{a, b\}: \min\{2 + \{b\}.cost\} = 4$$

$$\{c\}: \min\{1 + \{a, b\}.cost, 2 + \{a, c\}.cost, 2 + \{b, c\}.cost\} = 5$$

$$\{a, c\}: \min\{1 + \{a, b\}.cost, 2 + \{b, c\}.cost\} = 5$$

$$\{b, c\}: \min\{2 + \{a, c\}.cost, 2 + \{b, c\}.cost\} = 7$$

$$h^2(I, \{a, b, c\}) = \max\{h^2(I, \{a\}), h^2(I, \{b\}), h^2(I, \{c\})\}$$

$$h^2(I, \{a, b\}), h^2(I, \{a, c\}), h^2(I, \{b, c\})\}$$

$$= \max\{0, 2, 5, 4, 5, 7\} = 7$$

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Summary

- Critical path heuristic h^m estimates the cost of reaching a set ($\hat{=}$ conjunction) of variables as the cost of reaching the most expensive subset of size at most m .
- h^m computation is polynomial for fixed m .
- h^m computation is exponential in m .
- In practice, we use $m \in \{1, 2, 3\}$.