Planning and Optimization
D10. M&S: Merging Strategies and Label Reduction
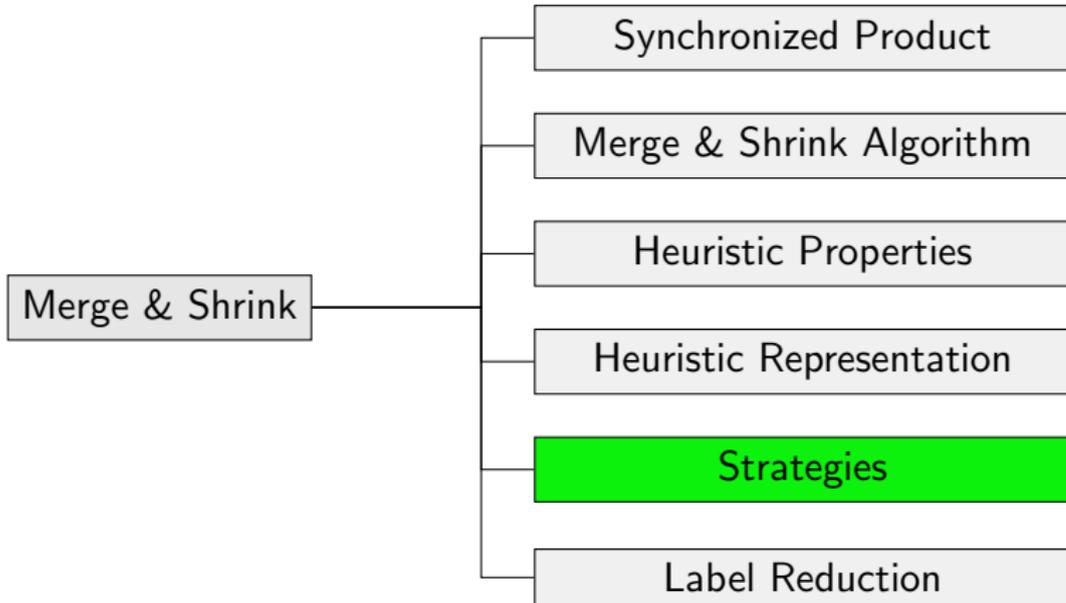
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Merging Strategies
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Label Reduction
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Merging Strategies

Merging Strategies

Generic Algorithm Template

Generic M&S computation algorithm

$abs := \{\mathcal{T}^{\pi\{v\}} \mid v \in V\}$
while abs contains more than one abstract transition system:
       select $\mathcal{A}_1$, $\mathcal{A}_2$ from abs
       shrink $\mathcal{A}_1$ and/or $\mathcal{A}_2$ until $size(\mathcal{A}_1) \cdot size(\mathcal{A}_2) \leq N$
       $abs := abs \setminus \{\mathcal{A}_1, \mathcal{A}_2\} \cup \{\mathcal{A}_1 \otimes \mathcal{A}_2\}$
return the remaining abstract transition system in abs

Remaining question:

- Which abstractions to select? ⤳ merging strategy

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Linear Merging Strategies

Linear Merging Strategy

In each iteration after the first, choose the abstraction computed in the previous iteration as $\mathcal{A}_1$.

Rationale: only maintains one "complex" abstraction at a time

⤳ Fully defined by an ordering of atomic projections.

Linear Merging Strategies: Choosing the Ordering

Use similar causal graph criteria as for growing patterns.

Example: Strategy of h_{HHH}

h_{HHH}: Ordering of atomic projections

- Start with a goal variable.
- Add variables that appear in preconditions of operators affecting previous variables.
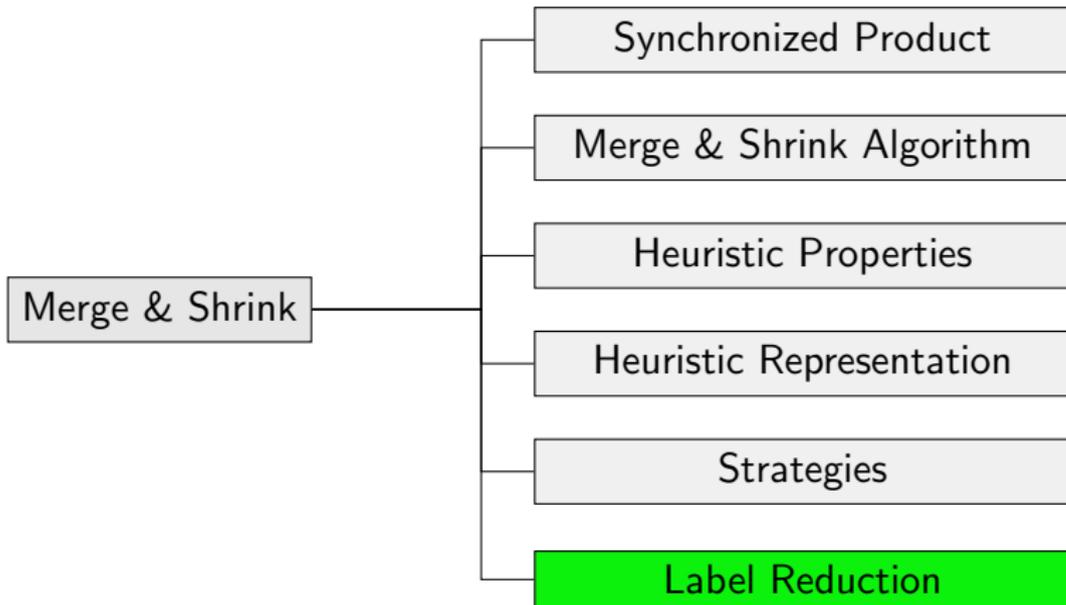- If that is not possible, add a goal variable.

Rationale: increases h quickly

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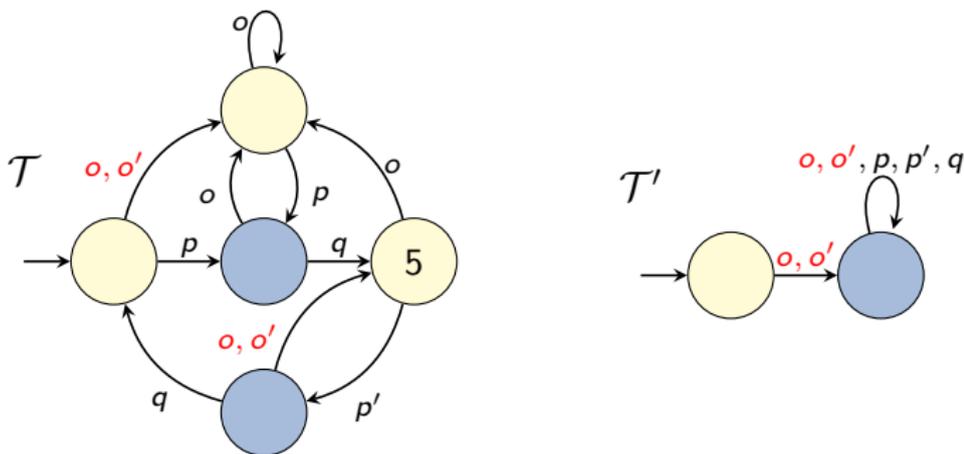○○○○

Non-linear Merging Strategies

- Non-linear merging strategies only recently gained more interest in the planning community.
- One reason: Better label reduction techniques (later in this chapter) enabled a more efficient computation.
- Examples:
    - DFP: preferably merge transition systems that must synchronize on labels that occur close to a goal state.
    - UMC and MIASM: Build clusters of variables with strong interactions and first merge variables within each cluster.
- Each merge-and-shrink heuristic computed with a non-linear merging strategy can also be computed with a linear merging strategy.
- However, linear merging can require a super-polynomial blow-up of the final representation size.

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Label Reduction

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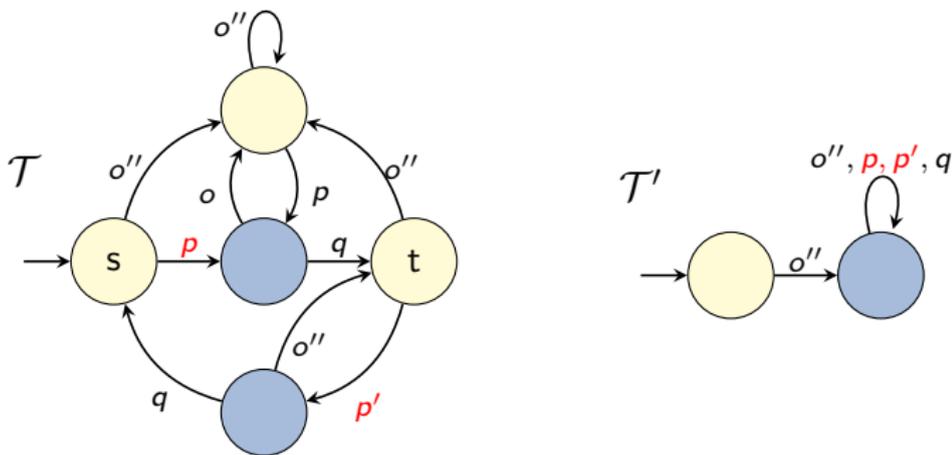oo●ooooooooooooo

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Label Reduction: Motivation (1)

Whenever there is a transition with label o' there is also a transition with label o. If o' is not cheaper than o, we can always use the transition with o.

Idea: Replace o and o' with label o'' with cost of o

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Label Reduction: Motivation (2)

States s and t are not bisimilar due to labels p and p'. In $\mathcal{T}'$ they label the same (parallel) transitions. If p and p' have the same cost, in such a situation there is no need for distinguishing them.

Idea: Replace p and p' with label p'' with same cost.

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Label Reduction: Motivation (3)
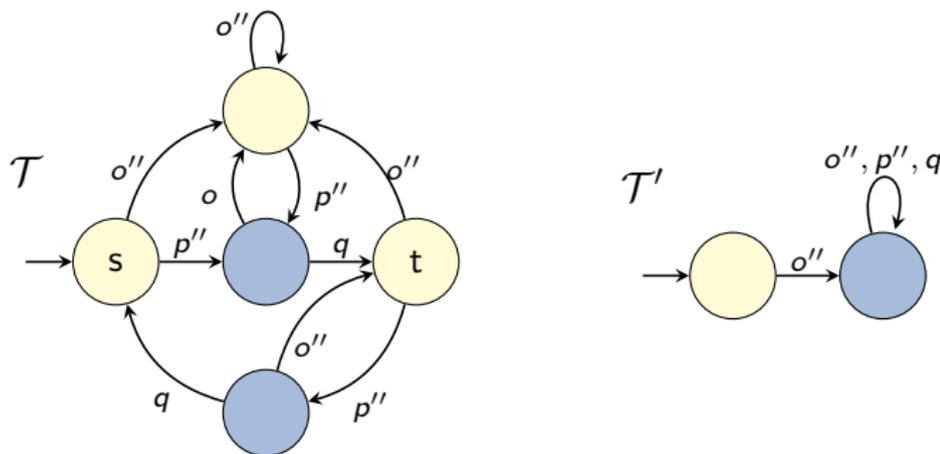
Label reductions reduce the time and memory requirement for merge and shrink steps and enable coarser bisimulation abstractions.

When is label reduction a safe transformation?

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Label Reduction: Definition

Definition (Label Reduction)

Let X be a collection of transition systems with label set L and label cost function c. A label reduction $\langle \lambda, c' \rangle$ for X is given by a function $\lambda : L \to L'$, where L' is an arbitrary set of labels, and a label cost function c' on L' such that for all $\ell \in L$, $c'(\lambda(\ell)) \leq c(\ell)$.

For $\mathcal{T} = \langle S, L, c, T, s_0, S_\star \rangle \in X$ the label-reduced transition system is $\mathcal{T}^{\langle \lambda, c' \rangle} = \langle S, L', c', \{\langle s, \lambda(\ell), t \rangle \mid \langle s, \ell, t \rangle \in T\}, s_0, S_\star \rangle$.

The label-reduced collection is $X^{\langle \lambda, c' \rangle} = \{\mathcal{T}^{\langle \lambda, c' \rangle} \mid \mathcal{T} \in X\}$.

$L' \cap L \neq \emptyset$ and $L' = L$ are allowed.

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Label Reduction is Safe (1)

Theorem (Label Reduction is Safe)

Let X be a collection of transition systems and $\langle \lambda, c' \rangle$ be a label-reduction for X. The transformation from X to $X^{\langle \lambda, c' \rangle}$ is safe.

Proof.

We show that the transformation is safe, using $\sigma = \text{id}$ for the mapping of states and λ for the mapping of labels.

The label cost function of $\mathcal{T}_{X^{\langle \lambda, c' \rangle}}$ is c' and has the required property by the definition of label reduction.   . . .

Label Reduction is Safe (2)

Theorem (Label Reduction is Safe)

Let X be a collection of transition systems and $\langle \lambda, c' \rangle$ be a label-reduction for X. The transformation from X to $X^{\langle \lambda, c' \rangle}$ is safe.

Proof (continued).

By the definition of synchronized products, $\mathcal{T}_X$ has a transition $\langle \langle s_1, \ldots, s_{|X|} \rangle, \ell, \langle t_1, \ldots, t_{|X|} \rangle \rangle$ if for all i, $\mathcal{T}_i \in X$ has a transition $\langle s_i, \ell, t_i \rangle$. By the definition of label-reduced transition systems, this implies that $\mathcal{T}^{\langle \lambda, c' \rangle}$ has a corresponding transition $\langle s_i, \lambda(\ell), t_i \rangle$, so $\mathcal{T}_{X^{\langle \lambda, c' \rangle}}$ has a transition $\langle s, \lambda(\ell), t \rangle = \langle \sigma(s), \lambda(\ell), \sigma(t) \rangle$ (definition of synchronized products).

For each goal state $s_\star$ of $\mathcal{T}_X$, state $\sigma(s_\star) = s_\star$ is a goal state of $\mathcal{T}_{X^{\langle \lambda, c' \rangle}}$ because the transformation replaces each transition system with a system that has the same goal states. $\qquad\square$

More Terminology

Let X be a collection of transition systems with labels L. Let $\ell, \ell' \in L$ be labels and let $\mathcal{T} \in X$.

- Label ℓ is alive in X if all $\mathcal{T}' \in X$ have some transition labelled with ℓ. Otherwise, ℓ is dead.
- Label ℓ locally subsumes label ℓ' in $\mathcal{T}$ if for all transitions $\langle s, \ell', t \rangle$ of $\mathcal{T}$ there is also a transition $\langle s, \ell, t \rangle$ in $\mathcal{T}$.
- ℓ globally subsumes ℓ' if it locally subsumes ℓ' in all $\mathcal{T}' \in X$.
- ℓ and ℓ' are locally equivalent in $\mathcal{T}$ if they label the same transitions in $\mathcal{T}$, i.e. ℓ locally subsumes ℓ' in $\mathcal{T}$ and vice versa.
- ℓ and ℓ' are $\mathcal{T}$-combinable if they are locally equivalent in all transition systems $\mathcal{T}' \in X \setminus \{\mathcal{T}\}$.

Exact Label Reduction

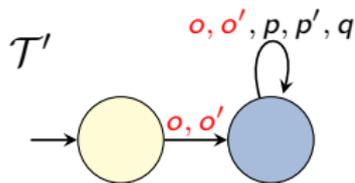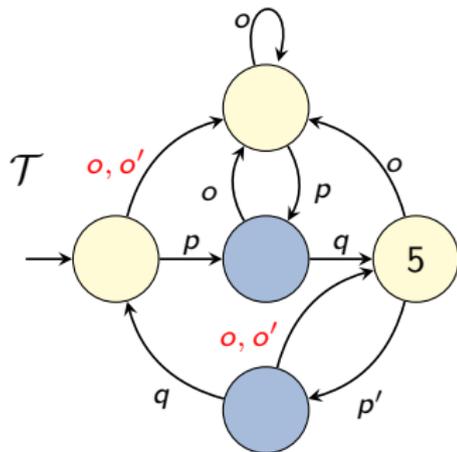> **Theorem (Criteria for Exact Label Reduction)**
>
> *Let X be a collection of transition systems with cost function c and label set L that contains no dead labels.*
>
> *Let $\langle \lambda, c' \rangle$ be a label-reduction for X such that λ combines labels ℓ_1 and ℓ_2 and leaves other labels unchanged. The transformation from X to $X^{\langle \lambda, c' \rangle}$ is exact iff $c(\ell_1) = c(\ell_2)$, $c'(\lambda(\ell)) = c(\ell)$ for all $\ell \in L$, and*
>
> - *ℓ_1 globally subsumes ℓ_2, or*
> - *ℓ_2 globally subsumes ℓ_1, or*
> - *ℓ_1 and ℓ_2 are $\mathcal{T}$-combinable for some $\mathcal{T} \in X$.*

(Proof omitted.)

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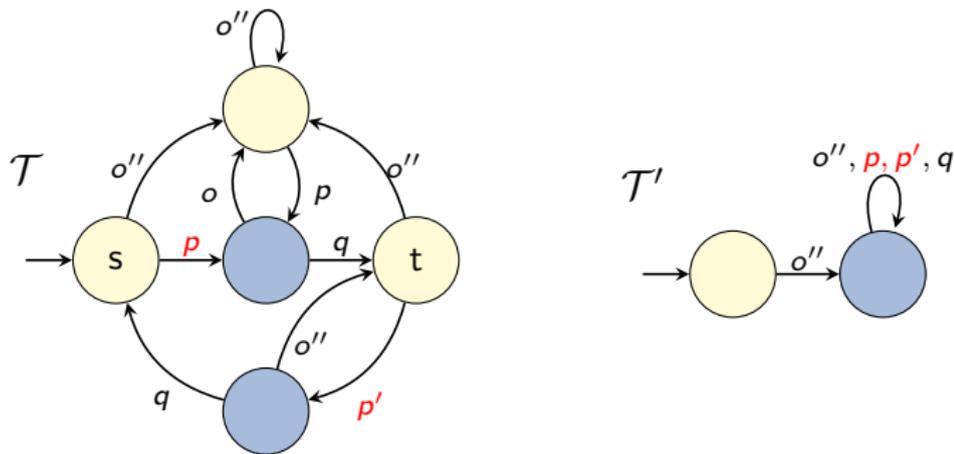
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Back to Example (1)

Label o' globally subsumes label o.

Back to Example (2)

Labels p and p' are $\mathcal{T}$-combinable.

Computation of Exact Label Reduction (1)

- For given labels ℓ_1, ℓ_2, the criteria can be tested in low-order polynomial time.
- Finding globally subsumed labels involves finding subset relationsships in a set family.
  ↝ no linear-time algorithms known
- The following algorithm exploits only $\mathcal{T}$-combinability.

Computation of Exact Label Reduction (2)

$eq_i :=$ set of label equivalence classes of $\mathcal{T}_i \in X$

Label-reduction based on $\mathcal{T}_i$-combinability

$eq := \{L\}$
for $j \in \{1, \ldots, |X|\} \setminus \{i\}$
    Refine eq with eq_j
// two labels are in the same set of eq
// iff they are locally equivalent in all $\mathcal{T}_j \neq \mathcal{T}_i$.
$\lambda = \text{id}$
for $B \in eq$
    $samecost := \{[\ell]_{\sim_c} \mid \ell \in B, \ell' \sim_c \ell'' \text{ iff } c(\ell') = c(\ell'')\}$
    for $L' \in samecost$
        $\ell_{\text{new}} := \text{new label}$
        $c'(\ell_{\text{new}}) := \text{cost of labels in } L'$
        for $\ell \in L'$
            $\lambda(\ell) = \ell_{\text{new}}$

Application in Merge-and-Shrink Algorithm

Generic M&S Computation Algorithm with Label Reduction

$abs := \{\mathcal{T}^{\pi_{\{v\}}} \mid v \in V\}$

while abs contains more than one abstract transition system:

       select $\mathcal{T}_1$, $\mathcal{T}_2$ from abs

       possibly label-reduce all $\mathcal{T} \in abs$

         (e.g. based on $\mathcal{T}_1$- and/or $\mathcal{T}_2$-combinability).

       shrink $\mathcal{T}_1$ and/or $\mathcal{T}_2$ until $size(\mathcal{T}_1) \cdot size(\mathcal{T}_2) \leq N$

       possibly label-reduce all $\mathcal{T} \in abs$

       $abs := abs \setminus \{\mathcal{T}_1, \mathcal{T}_2\} \cup \{\mathcal{T}_1 \otimes \mathcal{T}_2\}$

return the remaining abstract transition system in abs

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Summary

- There is a wide range of merging strategies. We only covered some important ones.
- Label reduction is crucial for the performance of the merge-and-shrink algorithm, especially when using bisimilarity for shrinking.

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Literature

Literature (1)

References on merge-and-shrink abstractions:

📄 Klaus Dräger, Bernd Finkbeiner and Andreas Podelski.
Directed Model Checking with Distance-Preserving
Abstractions.
Proc. SPIN 2006, pp. 19–34, 2006.
Introduces merge-and-shrink abstractions (for model-checking)
and DFP merging strategy.

📄 Malte Helmert, Patrik Haslum and Jörg Hoffmann.
Flexible Abstraction Heuristics for Optimal Sequential
Planning.
Proc. ICAPS 2007, pp. 176–183, 2007.
Introduces merge-and-shrink abstractions for planning.

Literature (2)

📄 Raz Nissim, Jörg Hoffmann and Malte Helmert.
Computing Perfect Heuristics in Polynomial Time: On
Bisimulation and Merge-and-Shrink Abstractions in Optimal
Planning.
Proc. IJCAI 2011, pp. 1983–1990, 2011.
Introduces bisimulation-based shrinking.

📄 Malte Helmert, Patrik Haslum, Jörg Hoffmann and Raz
Nissim.
Merge-and-Shrink Abstraction: A Method for Generating
Lower Bounds in Factored State Spaces.
Journal of the ACM 61 (3), pp. 16:1–63, 2014.
Detailed journal version of the previous two publications.

Literature (3)

📄 Silvan Sievers, Martin Wehrle and Malte Helmert.
Generalized Label Reduction for Merge-and-Shrink Heuristics.
Proc. AAAI 2014, pp. 2358–2366, 2014.
Introduces label reduction as covered in these slides
(there has been a more complicated version before).

📄 Gaojian Fan, Martin Müller and Robert Holte.
Non-linear merging strategies for merge-and-shrink based on
variable interactions.
Proc. AAAI 2014, pp. 2358–2366, 2014.
Introduces UMC and MIASM merging strategies