Planning and Optimization

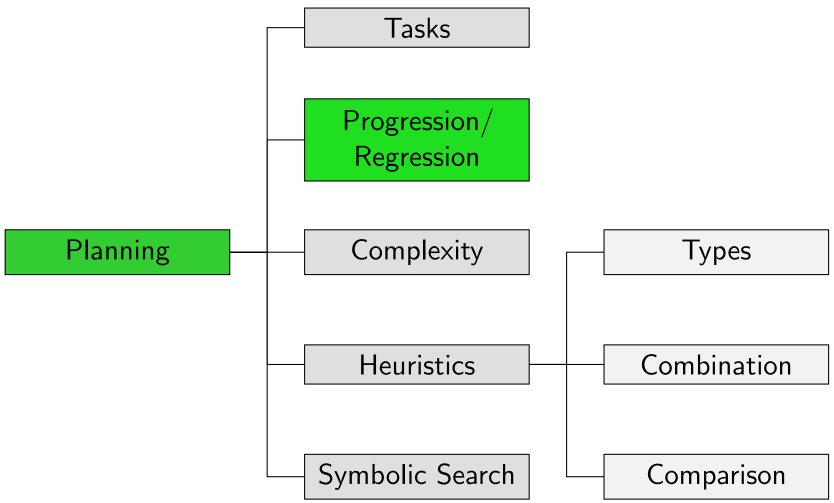
B3. General Regression, Part I

Malte Helmert and Gabriele Röger

Universität Basel

October 16, 2017

Content of this Course



Regression for General Planning Tasks

- With disjunctions and conditional effects, things become more tricky. How to regress $a \lor (b \land c)$ with respect to $\langle q, d \rhd b \rangle$?
- In this chapter, we show how to regress general sets of states through general operators.
- We extensively use the idea of representing sets of states as formulas.

B3.1 Regressing State Variables

Regressing State Variables: Motivation

Key question for general regression:

- Assume we are applying an operator with effect e.
- What must be true in the predecessor state for state variable v to be true in the successor state?

If we can answer this question, a general definition of regression is only a small additional step.

Regressing State Variables: Key Idea

Assume we are in state s and apply effect e to obtain successor state s'.

State variable v is true in s' if

- effect e makes it true, or
- it remains true, i.e., it is true in s and not made false by e.

Regressing a State Variable Through an Effect

Definition (Regressing a State Variable Through an Effect)

Let e be an effect of a propositional planning task, and let v be a state variable.

The regression of v through e, written $regr(v, e)$, is defined as the following logical formula:

$$regr(v, e) = effcond(v, e) \lor (v \land \neg effcond(\neg v, e)).$$

Question: Does this capture add-after-delete semantics correctly?

Regressing State Variables: Example

Example

Let $e = (b \rhd a) \land (c \rhd \neg a) \land b \land \neg d$.

v	$regr(v, e)$
a	$b \lor (a \land \neg c)$
b	$\top \lor (b \land \neg\bot) \equiv \top$
c	$\bot \lor (c \land \neg\bot) \equiv c$
d	$\bot \lor (d \land \neg\top) \equiv \bot$

Reminder: $regr(v, e) = effcond(v, e) \lor (v \land \neg effcond(\neg v, e))$

Regressing State Variables: Correctness (1)

Lemma (Correctness of $regr(v, e)$)

In a propositional planning task, let v be a state variable, o an operator and s a state in which o is applicable.

Then $s \models regr(v, eff(o))$ iff $s[\![o]\!] \models v$.

Regressing State Variables: Correctness (2)

Proof.

Let $e = eff(o)$.

($\Rightarrow$): We know $s \models regr(v, e)$, and hence $s \models effcond(v, e) \lor (v \land \neg effcond(\neg v, e))$.

Do a case analysis on the two disjuncts.

Case 1: $s \models effcond(v, e)$.
Then $s[\![o]\!] \models v$ by the first case in the def. of $s[\![o]\!]$ (Chapter A4).

Case 2: $s \models (v \land \neg effcond(\neg v, e))$.
Then $s \models v$ and $s \not\models effcond(\neg v, e)$.
We may additionally assume $s \not\models effcond(v, e)$ because otherwise we can apply Case 1 of this proof.
Then $s[\![o]\!] \models v$ by the third case in the def. of $s[\![o]\!]$ (Chapter A4).

...

Regressing State Variables: Correctness (3)

Proof (continued).

($\Leftarrow$): Proof by contraposition.
We show that if $regr(v, e)$ is false in s, then v is false in $s[\![o]\!]$.

- By prerequisite, $s \not\models effcond(v, e) \lor (v \land \neg effcond(\neg v, e))$.
- Hence $s \models \neg effcond(v, e) \land (\neg v \lor effcond(\neg v, e))$.
- From the first conjunct, we get $s \models \neg effcond(v, e)$ and hence $s \not\models effcond(v, e)$.
- From the second conjunct, we get $s \models \neg v \lor effcond(\neg v, e)$.
- Case 1: $s \models \neg v$. Then v is false before applying o and remains false, so $s[\![o]\!] \not\models v$.
- Case 2: $s \models effcond(\neg v, e)$. Then v is deleted by o and not simultaneously added, so $s[\![o]\!] \not\models v$.

□

B3.2 Regressing Formulas Through Effects

Regressing Formulas Through Effects: Idea

- We can now generalize regression from state variables to general formulas over state variables.
- The basic idea is to replace every occurrence of every state variable v by $regr(v, e)$ as defined in the previous section.
- The following definition makes this more formal.

Regressing Formulas Through Effects: Definition

Definition (Regressing a Formula Through an Effect)

In a propositional planning task, let e be an effect, and let φ be a formula over state variables.

The regression of φ through e, written $regr(\varphi, e)$, is defined as the following logical formula:

$$
\begin{aligned}
regr(\top, e) &= \top \\
regr(\bot, e) &= \bot \\
regr(v, e) &= effcond(v, e) \lor (v \land \neg effcond(\neg v, e)) \\
regr(\neg\psi, e) &= \neg regr(\psi, e) \\
regr(\psi \lor \chi, e) &= regr(\psi, e) \lor regr(\chi, e) \\
regr(\psi \land \chi, e) &= regr(\psi, e) \land regr(\chi, e).
\end{aligned}
$$

Regressing Formulas Through Effects: Example

Example

Let $e = (b \rhd a) \land (c \rhd \neg a) \land b \land \neg d$.

Recall:

- $regr(a, e) \equiv b \lor (a \land \neg c)$
- $regr(b, e) \equiv \top$
- $regr(c, e) \equiv c$
- $regr(d, e) \equiv \bot$

We get:

$$
\begin{aligned}
regr((a \lor d) \land (c \lor d), e) &\equiv ((b \lor (a \land \neg c)) \lor \bot) \land (c \lor \bot) \\
&\equiv (b \lor (a \land \neg c)) \land c \\
&\equiv b \land c
\end{aligned}
$$

Regressing Formulas Through Effects: Correctness (1)

Lemma (Correctness of $regr(\varphi, e)$)

Let φ be a logical formula, o an operator and s a state in which o is applicable.

Then $s \models regr(\varphi, eff(o))$ iff $s[\![o]\!] \models \varphi$.

Regressing Formulas Through Effects: Correctness (2)

Proof.

The proof is by structural induction on φ.

Induction hypothesis: $s \models regr(\psi, eff(o))$ iff $s[\![o]\!] \models \psi$ for all proper subformulas ψ of φ.

Base case $\varphi = \top$:

We have $regr(\top, eff(o)) = \top$, and $s \models \top$ iff $s[\![o]\!] \models \top$ is correct.

Base case $\varphi = \bot$:

We have $regr(\bot, eff(o)) = \bot$, and $s \models \bot$ iff $s[\![o]\!] \models \bot$ is correct.

Base case $\varphi = v$:

We have $s \models regr(v, eff(o))$ iff $s[\![o]\!] \models v$ from the previous lemma.

...

Regressing Formulas Through Effects: Correctness (3)

Proof (continued).

Inductive case $\varphi = \neg\psi$:

$$\begin{aligned} s \models regr(\neg\psi, eff(o)) &\text{ iff } s \models \neg regr(\psi, eff(o)) \\ &\text{ iff } s \not\models regr(\psi, eff(o)) \\ &\text{ iff } s[\![o]\!] \not\models \psi \\ &\text{ iff } s[\![o]\!] \models \neg\psi \end{aligned}$$

Inductive case $\varphi = \psi \lor \chi$:

$$\begin{aligned} s \models regr(\psi \lor \chi, eff(o)) &\text{ iff } s \models regr(\psi, eff(o)) \lor regr(\chi, eff(o)) \\ &\text{ iff } s \models regr(\psi, eff(o)) \text{ or } s \models regr(\chi, eff(o)) \\ &\text{ iff } s[\![o]\!] \models \psi \text{ or } s[\![o]\!] \models \chi \\ &\text{ iff } s[\![o]\!] \models \psi \lor \chi \end{aligned}$$

Inductive case $\varphi = \psi \land \chi$:

Like previous case, replacing "$\lor$" by "$\land$" and replacing "or" by "and". □

B3.3 Summary

Summary

- Regressing a state variable through an (arbitrary) operator must consider two cases:
  - state variables made true (by add effects)
  - state variables remaining true (by absence of delete effects)
- Regression of state variables can be generalized to arbitrary formulas φ by replacing each occurrence of a state variable in φ by its regression.