# Planning and Optimization

## A8. Finite Domain Representation

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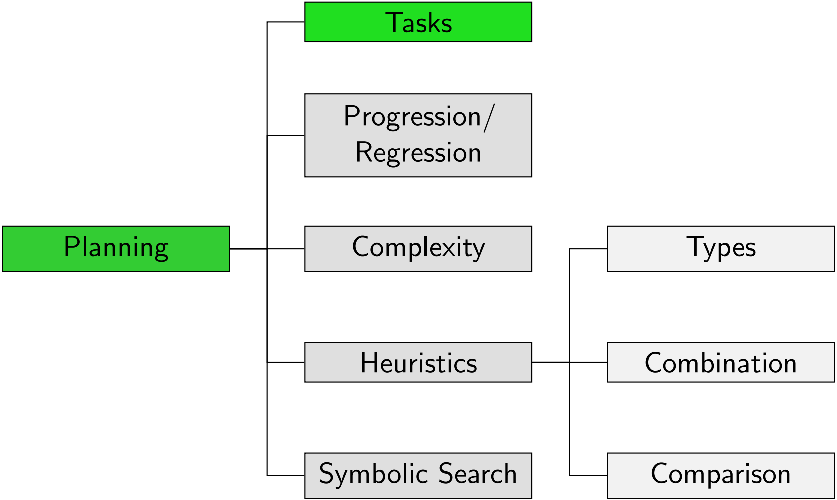
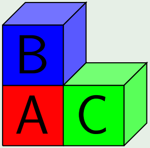
# Content of this Course



# Reminder: Blocks World with Boolean State Variables

## Example

$$s(A\text{-}on\text{-}B) = \mathbf{F}$$
$$s(A\text{-}on\text{-}C) = \mathbf{F}$$
$$s(A\text{-}on\text{-}table) = \mathbf{T}$$
$$s(B\text{-}on\text{-}A) = \mathbf{T}$$
$$s(B\text{-}on\text{-}C) = \mathbf{F}$$
$$s(B\text{-}on\text{-}table) = \mathbf{F}$$
$$s(C\text{-}on\text{-}A) = \mathbf{F}$$
$$s(C\text{-}on\text{-}B) = \mathbf{F}$$
$$s(C\text{-}on\text{-}table) = \mathbf{T}$$




$\rightsquigarrow 2^9 = 512$ states

Note: it may be useful to add auxiliary state variables like *A-clear*.

# Blocks World with Finite-Domain State Variables

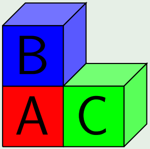
Use three finite-domain state variables:

- *below-a*: $\{\mathrm{b}, \mathrm{c}, \mathrm{table}\}$
- *below-b*: $\{\mathrm{a}, \mathrm{c}, \mathrm{table}\}$
- *below-c*: $\{\mathrm{a}, \mathrm{b}, \mathrm{table}\}$

## Example

$$s(\textit{below-a}) = \mathrm{table}$$
$$s(\textit{below-b}) = \mathrm{a}$$
$$s(\textit{below-c}) = \mathrm{table}$$

$\rightsquigarrow 3^3 = 27$ states




Note: it may be useful to add auxiliary state variables like *above-a*.

# FDR Planning Tasks

# Finite-Domain State Variables

## Definition (Finite-Domain State Variable)

A finite-domain state variable is a symbol $v$
with an associated finite domain, i.e., a non-empty finite set.

We write $\mathrm{dom}(v)$ for the domain of $v$.

## Example (Blocks World)

$v = \mathit{above\text{-}a}$, $\mathrm{dom}(\mathit{above\text{-}a}) = \{\mathrm{b}, \mathrm{c}, \mathrm{nothing}\}$

This state variable encodes the same information as the propositional variables *B-on-A*, *C-on-A* and *A-clear*.

# Finite-Domain States

## Definition (Finite-Domain State)

Let $V$ be a finite set of finite-domain state variables.

A state over $V$ is an assignment $s : V \to \bigcup_{v \in V} \mathrm{dom}(v)$ such that $s(v) \in \mathrm{dom}(v)$ for all $v \in V$.

## Example (Blocks World)

$$s = \{\textit{above-a} \mapsto \text{nothing}, \textit{above-b} \mapsto \text{a}, \textit{above-c} \mapsto \text{b}, \textit{below-a} \mapsto \text{b}, \textit{below-b} \mapsto \text{c}, \textit{below-c} \mapsto \text{table}\}$$

# Finite-Domain Formulas

## Definition (Finite-Domain Formula)

Logical formulas over finite-domain state variables $V$ are defined identically to the propositional case, except that instead of atomic formulas of the form $v' \in V'$ with propositional state variables $V'$, there are atomic formulas of the form $v = d$, where $v \in V$ and $d \in \mathrm{dom}(v)$.

## Example (Blocks World)

The formula $(\mathit{above\text{-}a} = \mathrm{nothing}) \lor \neg(\mathit{below\text{-}b} = \mathrm{c})$ corresponds to the formula $\mathit{A\text{-}clear} \lor \neg \mathit{B\text{-}on\text{-}C}$.

# Finite-Domain Effects

**Definition (Finite-Domain Effect)**

Effects over finite-domain state variables $V$
are defined identically to the propositional case,
except that instead of atomic effects of the form $v'$ and $\neg v'$
with propositional state variables $v' \in V'$, there are atomic effects
of the form $v := d$, where $v \in V$ and $d \in \mathrm{dom}(v)$.

**Example (Blocks World)**

The effect
$(\textit{below-a} := \mathrm{table}) \wedge ((\textit{above-b} = \mathrm{a}) \rhd (\textit{above-b} := \mathrm{nothing}))$
corresponds to the effect
$\textit{A-on-table} \wedge \neg \textit{A-on-B} \wedge \neg \textit{A-on-C} \wedge (\textit{A-on-B} \rhd (\textit{B-clear} \wedge \neg \textit{A-on-B}))$.

$\rightsquigarrow$ finite-domain operators, effect conditions etc. follow

# Planning Tasks in Finite-Domain Representation

## Definition (Planning Task in Finite-Domain Representation)

A planning task in finite-domain representation
or FDR planning task is a 4-tuple $\Pi = \langle V, I, O, \gamma \rangle$ where

- $V$ is a finite set of finite-domain state variables,
- $I$ is a state over $V$ called the initial state,
- $O$ is a finite set of finite-domain operators over $V$, and
- $\gamma$ is a formula over $V$ called the goal.

# FDR Task Semantics

# FDR Task Semantics: Introduction

- We have now defined what FDR tasks look like.
- We still have to define their semantics.
- Because they are similar to propositional planning tasks, we can define their semantics in a very similar way.

# Direct vs. Compilation Semantics

We describe two ways of defining semantics for FDR tasks:

- directly, mirroring our definitions for propositional tasks
- by compilation to propositional tasks

Comparison of the semantics:

- The two semantics are equivalent in terms of the reachable state space and hence in terms of the set of solutions. (We will not prove this.)
- They are not equivalent w.r.t. the set of all states.

Where the distinction matters, we use the direct semantics in this course unless stated otherwise.

# Conflicting Effects

- As with propositional planning tasks, there is a subtlety: what should an effect of the form $v := a \wedge v := b$ mean?
- For FDR tasks, the common convention is to make this illegal, i.e., to make an operator inapplicable if it would lead to conflicting effects.

# Consistency Condition and Applicability

## Definition (Consistency Condition)

Let $e$ be an effect over finite-domain state variables $V$.

The consistency condition for $e$, $consist(e)$ is defined as

$$\bigwedge_{v \in V} \bigwedge_{d, d' \in \mathrm{dom}(v), d \neq d'} \neg(effcond(v := d, e) \wedge effcond(v := d', e)).$$

## Definition (Applicable FDR Operator)

An FDR operator $o$ is applicable in a state $s$
if $s \models pre(o) \wedge consist(eff(o))$.

The definitions of $s[\![o]\!]$ etc. then follow in the natural way.

# Reminder: Semantics of Propositional Planning Tasks

Reminder from Chapter A4:

**Definition (Transition System Induced by a Prop. Planning Task)**

The propositional planning task $\Pi = \langle V, I, O, \gamma \rangle$ induces the transition system $\mathcal{T}(\Pi) = \langle S, L, c, T, s_0, S_\star \rangle$, where

- $S$ is the set of all valuations of $V$,
- $L$ is the set of operators $O$,
- $c(o) = \mathit{cost}(o)$ for all operators $o \in O$,
- $T = \{\langle s, o, s' \rangle \mid s \in S,\ o \text{ applicable in } s,\ s' = s[\![o]\!]\}$,
- $s_0 = I$, and
- $S_\star = \{s \in S \mid s \models \gamma\}$.

# Semantics of Planning Tasks

A definition that works for both types of planning tasks:

**Definition (Transition System Induced by a Planning Task)**

The planning task $\Pi = \langle V, I, O, \gamma \rangle$ induces
the transition system $\mathcal{T}(\Pi) = \langle S, L, c, T, s_0, S_\star \rangle$, where

- $S$ is the set of states over $V$,
- $L$ is the set of operators $O$,
- $c(o) = cost(o)$ for all operators $o \in O$,
- $T = \{\langle s, o, s' \rangle \mid s \in S,\ o \text{ applicable in } s,\ s' = s[\![o]\!]\}$,
- $s_0 = I$, and
- $S_\star = \{s \in S \mid s \models \gamma\}$.

Planning task here refers to either a propositional or FDR task.

# Compilation Semantics

## Definition (Induced Propositional Planning Task)

Let $\Pi = \langle V, I, O, \gamma \rangle$ be an FDR planning task.
The induced propositional planning task $\Pi'$ is the (regular) planning task $\Pi' = \langle V', I', O', \gamma' \rangle$, where

- $V' = \{\langle v, d \rangle \mid v \in V, d \in \text{dom}(v)\}$
- $I'(\langle v, d \rangle) = \mathbf{T}$ iff $I(v) = d$
- $O'$ and $\gamma'$ are obtained from $O$ and $\gamma$ by
  - replacing each operator precondition $pre(o)$ by $pre(o) \wedge consist(eff(o))$, and then
  - replacing each atomic formula $v = d$ by the proposition $\langle v, d \rangle$,
  - replacing each atomic effect $v := d$ by the effect $\langle v, d \rangle \wedge \bigwedge_{d' \in \text{dom}(v) \setminus \{d\}} \neg \langle v, d' \rangle$.

# $SAS^+$ Planning Tasks

# SAS$^+$ Planning Tasks

## Definition (SAS$^+$ Planning Task)

An FDR planning task $\Pi = \langle V, I, O, \gamma \rangle$ is called an SAS$^+$ planning task if

- there are no conditional effects in $O$, and
- all operator preconditions in $O$ and the goal formula $\gamma$ are conjunctions of atoms.

# SAS$^+$ vs. STRIPS

- SAS$^+$ is the analogue of STRIPS planning tasks for FDR
- induced propositional planning task of a SAS$^+$ task is a STRIPS planning task after simplification (consistency conditions simplify to $\bot$ or $\top$)
- FDR tasks obtained by mutex-based reformulation of STRIPS planning tasks are SAS$^+$ tasks

# Transition Normal Form

# Variables Occurring in Conditions and Effects

- Many algorithmic problems for SAS$^+$ planning tasks become simpler when we can make two further restrictions.
- These are related to the variables that occur in conditions and effects of the task.

**Definition ($vars(\varphi)$, $vars(e)$)**

For a logical formula $\varphi$ over finite-domain variables $V$, $vars(\varphi)$ denotes the set of finite-domain variables occurring in $\varphi$.

For an effect $e$ over finite-domain variables $V$, $vars(e)$ denotes the set of finite-domain variables occurring in $e$.

# Transition Normal Form

## Definition (Transition Normal Form)

A SAS$^+$ planning task $\Pi = \langle V, I, O, \gamma \rangle$ is in transition normal form (TNF) if

- for all $o \in O$, $vars(pre(o)) = vars(eff(o))$, and
- $vars(\gamma) = V$.

In words, an operator in TNF must mention the same variables in the precondition and effect, and a goal in TNF must mention all variables (= specify exactly one goal state).

# Converting Operators to TNF: Violations

There are two ways in which an operator $o$ can violate TNF:

- There exists a variable $v \in \mathit{vars}(\mathit{pre}(o)) \setminus \mathit{vars}(\mathit{eff}(o))$.
- There exists a variable $v \in \mathit{vars}(\mathit{eff}(o)) \setminus \mathit{vars}(\mathit{pre}(o))$.

The first case is easy to address: if $v = d$ is a precondition with no effect on $v$, just add the effect $v := d$.

The second case is more difficult: if we have the effect $v := d$ but no precondition on $v$, how can we add a precondition on $v$ without changing the meaning of the operator?

# Converting Operators to TNF: Multiplying Out

Solution 1: multiplying out

1. While there exists an operator $o$ and a variable $v \in vars(eff(o))$ with $v \notin vars(pre(o))$:
   - For each $d \in \text{dom}(v)$, add a new operator that is like $o$ but with the additional precondition $v = d$.
   - Remove the original operator.
2. Repeat the previous step until no more such variables exist.

Problem:

- If an operator $o$ has $n$ such variables, each with $k$ values in its domain, this introduces $k^n$ variants of $o$.
- Hence, this is an exponential transformation.

# Converting Operators to TNF: Auxiliary Values

## Solution 2: auxiliary values

1. For every variable $v$, add a new auxiliary value u to its domain.
2. For every variable $v$ and value $d \in \mathrm{dom}(v) \setminus \{\mathrm{u}\}$, add a new operator to change the value of $v$ from $d$ to u at no cost: $\langle v = d, v := \mathrm{u}, 0 \rangle$.
3. For all operators $o$ and all variables $v \in \mathit{vars}(\mathit{eff}(o)) \setminus \mathit{vars}(\mathit{pre}(o))$, add the precondition $v = \mathrm{u}$ to $\mathit{pre}(o)$.

## Properties:

- Transformation can be computed in linear time.
- Due to the auxiliary values, there are new states and transitions in the induced transition system, but all path costs between original states remain the same.

# Converting Goals to TNF

- The auxiliary value idea can also be used to convert the goal $\gamma$ to TNF.
- For every variable $v \notin vars(\gamma)$, add the condition $v = \mathrm{u}$ to $\gamma$.

With these ideas, every SAS$^+$ planning task can be converted into transition normal form in linear time.

# Summary

# Summary

- Planning tasks in finite-domain representation (FDR) are an alternative to propositional planning tasks.
- FDR tasks are often more compact (have fewer states).
- This makes many planning algorithms more efficient when working with a finite-domain representation.
- $SAS^+$ tasks are a restricted form of FDR tasks where only conjunctions of atoms are allowed in the preconditions, effects and goal. No conditional effects are allowed.
- Transition normal form (TNF) is even more restricted: for each operator, preconditions and effects must mention the same variables, and there must be a unique goal state.
- $SAS^+$ tasks can be converted to TNF in linear time.