Planning and Optimization

A5. Equivalent Operators and Effect Normal Form

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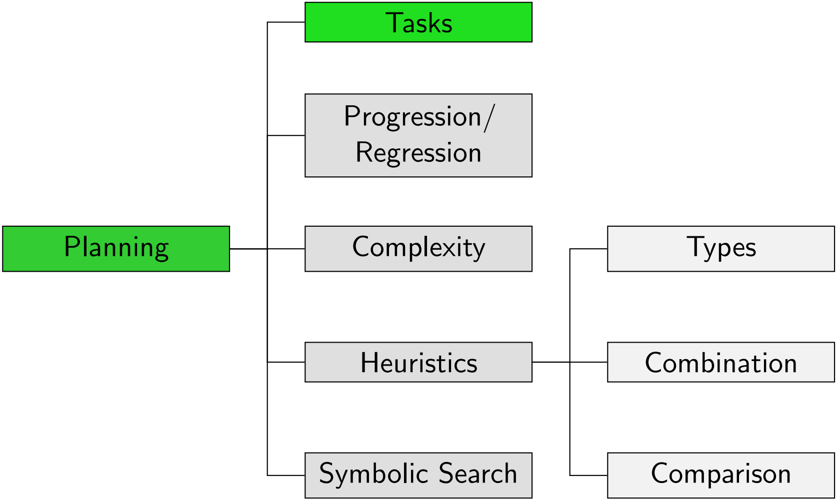
Content of this Course



Motivation

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Similarly to normal forms in propositional logic (DNF, CNF, NNF), we can define normal forms for effects, operators and propositional planning tasks.

This is useful because algorithms (and proofs) then only need to deal with effects, operators and tasks in normal form.

Equivalence Transformations

Equivalence of Operators and Effects: Definition

Definition (Equivalent Operators)

Two operators o and o' over state variables V are equivalent, written $o \equiv o'$, if $cost(o) = cost(o')$ and for all states s, s' over V, o induces the transition $s \xrightarrow{o} s'$ iff o' induces the transition $s \xrightarrow{o'} s'$.

Definition (Equivalent Effects)

Two effects e and e' over state variables V are equivalent, written $e \equiv e'$, if the operators $\langle \top, e, 0 \rangle$ and $\langle \top, e', 0 \rangle$ are equivalent.

Equivalence of Operators and Effects: Theorem

Theorem

Let o and o' be operators with $pre(o) \equiv pre(o')$, $eff(o) \equiv eff(o')$ and $cost(o) = cost(o')$. Then $o \equiv o'$.

Note: The converse is not true. (Why not?)

Equivalence Transformations for Effects

$$e_1 \wedge e_2 \equiv e_2 \wedge e_1 \tag{1}$$

$$(e_1 \wedge \cdots \wedge e_n) \wedge (e'_1 \wedge \cdots \wedge e'_m) \equiv e_1 \wedge \cdots \wedge e_n \wedge e'_1 \wedge \cdots \wedge e'_m \tag{2}$$

$$\top \wedge e \equiv e \tag{3}$$

$$\chi \rhd e \equiv \chi' \rhd e \qquad \text{if } \chi \equiv \chi' \tag{4}$$

$$\top \rhd e \equiv e \tag{5}$$

$$\bot \rhd e \equiv \top \tag{6}$$

$$\chi_1 \rhd (\chi_2 \rhd e) \equiv (\chi_1 \wedge \chi_2) \rhd e \tag{7}$$

$$\chi \rhd (e_1 \wedge \cdots \wedge e_n) \equiv (\chi \rhd e_1) \wedge \cdots \wedge (\chi \rhd e_n) \tag{8}$$

$$(\chi_1 \rhd e) \wedge (\chi_2 \rhd e) \equiv (\chi_1 \vee \chi_2) \rhd e \tag{9}$$

Conflict-Free Effects

Conflict-Freeness: Motivation

- The add-after-delete semantics makes effects like $(a \rhd c) \land (b \rhd \neg c)$ somewhat unintuitive to interpret.

$\rightsquigarrow$ What happens in states where $a \land b$ is true?

- It would be nicer if $\mathit{effcond}(\neg v, e)$ were always the condition under which e makes v false (but because of add-after-delete, it is not).

$\rightsquigarrow$ introduce a normal form where the "complicated case" of add-after-delete semantics never arises

Conflict-Free Effects

Definition (Conflict-Free)

An effect e is called conflict-free if $\mathit{effcond}(v, e) \land \mathit{effcond}(\neg v, e)$ is unsatisfiable for all state variables v.

Testing if Effects are Conflict-Free

- In general, testing whether an effect is conflict-free is a coNP-complete problem. (Why?)
- However, we do not usually need such a test. Instead, we can produce an equivalent conflict-free effect in polynomial time.
- Algorithm: given effect e, replace each atomic effect of the form $\neg v$ by $(\neg effcond(v, e) \rhd \neg v)$.
  The resulting effect e' is conflict-free and $e \equiv e'$. (Why?)

Effect Normal Form

Effect Normal Form: Motivation

- CNF and DNF limit the nesting of connectives in propositional logic.
- For example, a CNF formula is
  - a conjunction of 0 or more subformulas,
  - each of which is a disjunction of 0 or more subformulas,
  - each of which is a literal.
- Similarly, we can define a normal form that limits the nesting of effects.
- This is useful because we then do not have to consider arbitrarily structured effects, e.g., when representing them in a planning algorithm.

Effect Normal Form

Definition (Effect Normal Form)

An effect e is in normal form if it is:

- a conjunctive effect
- whose conjuncts are conditional effects
- whose subeffects are atomic effects, and
- no atomic effect occurs in e multiple times.

If e is also conflict-free, we say it is in conflict-free normal formal.

An operator o is in (conflict-free) effect normal form
if $\mathit{eff}(o)$ is in (conflict-free) normal form.

Note: non-conjunctive effects can be considered
as conjunctive effects with 1 conjunct

Effects in Conflict-Free Normal Form: Example

Example

Consider the effect

$$c \land (a \rhd (\neg b \land (c \rhd (b \land \neg d \land \neg a)))) \land (\neg b \rhd \neg a)$$

An equivalent effect in conflict-free normal form is

$$\begin{gathered}(\top \rhd c) \land \\ ((a \land \neg c) \rhd \neg b) \land \\ ((a \land c) \rhd b) \land \\ ((a \land c) \rhd \neg d) \land \\ ((\neg b \lor (a \land c)) \rhd \neg a)\end{gathered}$$

Note: for simplicity, we will often write $(\top \rhd \ell)$ as ℓ, i.e., omit trivial effect conditions. We will still consider such effects to be in normal form.

Producing Effects in Normal Form

Theorem

For every effect, an equivalent effect in normal form and an equivalent effect in conflict-free normal form can be computed in polynomial time.

Proof Sketch.

Every effect e over variables V is equivalent to
$\bigwedge_{v \in V}(\mathit{effcond}(v, e) \rhd v) \wedge \bigwedge_{v \in V}(\mathit{effcond}(\neg v, e) \rhd \neg v)$,
which is in normal form.

For conflict-free normal form, use $\mathit{effcond}(\neg v, e) \wedge \neg\mathit{effcond}(v, e)$ instead of $\mathit{effcond}(\neg v, e)$.

In both cases, conjuncts of the form $(\chi \rhd \ell)$ where $\chi \equiv \bot$ can be omitted to simplify the effect.

Summary

Summary

- Effect equivalences can be used to simplify operator effects.
- In conflict-free effects, the "complicated case" in the add-after-delete semantics of operators does not arise.
- For effects in normal form, the only permitted nesting is atomic effects within conditional effects within conjunctive effects, and all atomic effects must be distinct.
- For effects in conflict-free normal form, it is easy to determine the condition under which a given literal is made true by applying the effect in a given state.
- Every effect can be transformed into an equivalent effect in conflict-free normal form in polynomial time.