Planning and Optimization

E3. Symbolic Search: Uniform-cost and A* search

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Introduction

Introduction

- Previous chapter: Symbolic breadth-first search
- Optimal plans only guaranteed for unit-cost tasks (= all operators same cost)
- Optimal planning in explicit-state forward search:
  - (uninformed) uniform-cost search
  - (informed) A* search
  - ...

Analogous algorithms for symbolic (BDD-based) search?

Symbolic Uniform-Cost Search

Cost-separated Transition Relations

- Previously: one transition relation $T_V(O)$ for all operators
- Now: several transition relations for operators of same cost
- Set $\mathcal{T}$ of pairs (T, c), where T is a transition relation for one/some/all operators of cost c
  - All operators must be covered (and nothing else): $\bigcup_{(T,c)\in\mathcal{T}} r(T) = r(T_V(O))$
  - The cost must be correct: For $(T, c') \in \mathcal{T}$: if $a \in r(T)$ then $a \models \bigvee_{o\in O:cost(o)=c'} \tau_V(o)$

Many possibilities to split up $T_V(O)$ (discussed later)

Image Computation

- The apply function (previous chapter) computes the set of states S' that can be reached from a set of states S by applying one operator.
- This is called the image of S wrt. transition relation $T_V(O)$.
- Now: image computation for arbitrary transition relations.

def image(B, T):
  $B := \textit{bdd-intersection}(B, T)$
  for each $v \in V$:
    $B := \textit{bdd-forget}(B, v)$
  for each $v \in V$:
    $B := \textit{bdd-rename}(B, v', v)$
  return B

Exactly like apply but gets transition relation as argument.

Symbolic Uniform-Cost Search (Positive Operator Costs)

def symbolic-uniform-cost(V, I, O, γ):
  $goal := build\text{-}BDD(\gamma)$
  $\mathcal{T} := make\text{-}transition\text{-}relations(V, O)$
  $open_0 := bdd\text{-}state(I)$
  while $\exists g : open_g \neq \mathbf{0}$:
    $g := \min\{g \mid open_g \neq \mathbf{0}\}$
    $closed_g := open_g$
    if $bdd\text{-}intersection(open_g, goal) \neq \mathbf{0}$:
      return $construct\text{-}plan(I, \mathrm{O}, goal, closed_*, g)$
    for all $(T, c) \in \mathcal{T}$:
      $open_{g+c} := bdd\text{-}union(open_{g+c},$
        $image(open_g, T))$
    $open_g := \mathbf{0}$
  return unsolvable

Pre-image Computation

- The image of S wrt. transition relation T computes the set of states that can be reached from S by applying a transition represented by T.
- The pre-image of S wrt. T is the set of states from which we can reach S by applying a transition represented by T.

def pre-image(B, T):
  for each $v \in V$:
    $B := \textit{bdd-rename}(B, v, v')$
  $B := \textit{bdd-intersection}(B, T)$
  for each $v \in V$:
    $B := \textit{bdd-forget}(B, v')$
  return B

Plan Extraction (Positive Operator Costs)

```
def construct-plan(I, O, goal, closed_*, g):
    cut := bdd-intersection(goal, closed_g)
    init := bdd-state(I)
    π := ⟨⟩
    while bdd-intersection(cut, init) = 0:
        for o ∈ O:
            pre := pre-image(cut, τ_V(o))
            if c := bdd-intersection(pre, closed_{g−cost(o)}) ≠ 0:
                cut := c
                g := g − cost(o)
                π := ⟨o⟩π
                break
    return π
```

Zero-cost Operators

What is the problem with zero-cost operators?

- Search: could re-open $open_g$ after it was moved to $closed_g$, possibly running into an infinite loop
  $\rightarrow$ Apply all zero-cost operators before closing
- Plan extraction: could loop in zero-cost cycles
  $\rightarrow$ special treatment

Breadth-first Exploration with Zero-cost Operators

def bfs-zero(B, g, $\mathcal{T}$, $goal$):
  $i := 0$
  $closed_{g,i} := B$
  while $B \neq \mathbf{0}$ and $bdd\text{-}intersection(B, goal) = \mathbf{0}$:
    $B' := \mathbf{0}$
    for $(T, c) \in \mathcal{T}, c = 0$:
      $B' := bdd\text{-}union(B', image(B, T))$
    $B := bdd\text{-}intersection(B', bdd\text{-}complement(closed_{g,i}))$
    $i := i + 1$
    $closed_{g,i} := bdd\text{-}union(B, closed_{g,i-1})$
  return $closed_{g,i}$

Symbolic Uniform-Cost Search

```
def symbolic-uniform-cost(V, I, O, γ):
    goal := build-BDD(γ)
    𝒯 := make-transition-relations(V, O)
    open_0 := bdd-state(I)
    while ∃g : open_g ≠ 0:
        g := min{g | open_g ≠ 0}
        open_g := bfs-zero(open_g, g, 𝒯, goal)
        closed_g := open_g
        if bdd-intersection(open_g, goal) ≠ 0:
            return construct-plan(I, O, goal, closed_*, g)
        for all (T, c) ∈ 𝒯 with c > 0:
            open_{g+c} := bdd-union(open_{g+c},
                                    image(open_g, T))
        open_g := 0
    return unsolvable
```

Plan Extraction with Zero-cost Operators

Needs all closed sets form bfs-zero and symbolic-uniform-cost.

```
def construct-plan(I, O, goal, closed_{*,*}, g):
    cut := bdd-intersection(goal, closed_g)
    init := bdd-state(I); π := ⟨⟩
    while bdd-intersection(cut, init) = 0:
        cut, π := get-to-bfs-level-0(cut, g, closed_{g,*}, π, O)
        if g = 0:
            return π
        for o ∈ O with cost(o) > 0:
            pre := pre-image(cut, τ_V(o))
            if c := bdd-intersection(pre, closed_{g−cost(o)}) ≠ 0:
                cut := c; π := ⟨o⟩π
                g := g − cost(o)
                break
    return π
```

Plan Extraction: Zero-Cost Plan Fragment

def *get-to-bfs-level-0*$(cut, g, closed_{g,*}, \pi, O)$:
  $level := 0$
  while *bdd-intersection*$(cut, closed_{g,level}) = \mathbf{0}$:
    $level := level + 1$
  while $level \neq 0$:
    for $o \in O$ with $cost(o) = 0$:
      $pre :=$ *pre-image*$(cut, \tau_V(o))$
      if $c :=$ *bdd-intersection*$(pre, closed_{g,level-1}) \neq \mathbf{0}$:
        $cut := c$
        $level := level - 1$
        $\pi := \langle o \rangle \pi$
        break
  return cut, π

Pruning of Closed States

- In explicit-state uniform-cost search, we never re-expand closed states.
- We can easily introduce such pruning in symbolic uniform-cost search.

Uniform-Cost Search with Pruning of Closed States

```
def symbolic-uniform-cost(V, I, O, γ):
    goal := build-BDD(γ)
    𝒯 := make-transition-relations(V, O)
    open_0 := bdd-state(I)
    while ∃g : open_g ≠ 0:
        g := min{g | open_g ≠ 0}
        open_g := bfs-zero(open_g, g, 𝒯, goal, closed_*)
        closed_g := open_g
        if bdd-intersection(open_g, goal) ≠ 0:
            return construct-plan(I, O, goal, closed_*, g)
        for all (T, c) ∈ 𝒯 with c > 0:
            open_{g+c} := bdd-union(open_{g+c},
                                    image(open_g, T))
        open_g := 0
    return unsolvable
```

bfs-zero with Pruning of Closed States

```
def bfs-zero(B, g, T, goal, prune):
    for P ∈ prune:
        B := bdd-intersection(B, bdd-complement(P))
    i := 0
    closed_{g,i} := B
    while B ≠ 0 and bdd-intersection(B, goal) = 0:
        B' := 0
        for (T, c) ∈ T, c = 0:
            B' := bdd-union(B', image(B, T))
        B := bdd-intersection(B', bdd-complement(closed_{g,i}))
        for P ∈ prune:
            B := bdd-intersection(B, bdd-complement(P))
        i := i + 1
        closed_{g,i} := bdd-union(B, closed_{g,i-1})
    return closed_{g,i}
```

Symbolic A*

Symbolic A*

- Difference between explicit-state uniform-cost search and A*: heuristic to guide search
- $f = g + h$
- Analogously in symbolic search
- Heuristic given as set *heur* of BDDs $heur_h$ for each heuristic estimate h

Symbolic A* (with Consistent Heuristic)

def symbolic-AStar(V, I, O, γ, $heur$):

  $goal := build\text{-}BDD(\gamma)$

  $\mathcal{T} := make\text{-}transition\text{-}relations(V, O)$

  $open_{0,h(I)} := bdd\text{-}state(I)$

  while $\exists g, h : open_{g,h} \neq \mathbf{0}$:

    $f := \min\{f \mid \exists g, h : open_{g,h} \neq \mathbf{0}, f = g + h\}$

    $g := \min\{g \mid \exists h : open_{g,h} \neq \mathbf{0}, f = g + h\}$

    $open_{g,*} := expand_0(open_{*,*}, g, h, \mathcal{T}, goal, heur, closed_*)$

    $closed_g := bdd\text{-}union(closed_g, open_{g,h})$

    if $bdd\text{-}intersection(open_{g,h}, goal) \neq \mathbf{0}$:

      return $construct\text{-}plan(I, O, goal, closed_*, g)$

    $open_{*,*} := expand_{>0}(open_{*,*}, g, h, \mathcal{T}, heur)$

    $open_{g,h} := \mathbf{0}$

  return unsolvable

Symbolic A* (with Consistent Heuristic)

```
def symbolic-AStar(V, I, O, γ, heur):
    goal := build-BDD(γ)
    T := make-transition-relations(V, O)
    open_{0,h(I)} := bdd-state(I)
    while ∃g, h : open_{g,h} ≠ 0:
        f := min{f | ∃g, h : open_{g,h} ≠ 0, f = g + h}
        g := min{g | ∃h : open_{g,h} ≠ 0, f = g + h}
        open_{g,*} := expand_0(open_{*,*}, g, h, T, goal, heur, closed_*)
        closed_g := bdd-union(closed_g, open_{g,h})
        if bdd-intersection(open_{g,h}, goal) ≠ 0:
            return construct-plan(I, O, goal, closed_*, g)
        open_{*,*} := expand_{>0}(open_{*,*}, g, h, T, heur)
        open_{g,h} := 0
    return unsolvable
```

For performance it is important to expand the minimum g value.

Expand States and Update Open Lists

def expand$_0$($open_{*,*}$, g, h, $\mathcal{T}$, $goal$, $heur$, $prune$):
  $B := bfs\text{-}zero(open_{g,h}, (g, h), \mathcal{T}, goal, prune)$
  for $heur_{h'} \in heur, h \leq h' < \infty$:
    $B' := bdd\text{-}intersection(heur_{h'}, open\text{-}zero)$
    $open_{g,h'} := bdd\text{-}union(open_{g,h'}, B')$
  return $open_{g,*}$

def expand$_{>0}$($open_{*,*}$, g, h, $\mathcal{T}$, $heur$):
  for all $(T, c) \in \mathcal{T}, c > 0$:
    $B := image(open_{g,h}, T)$
  for $heur_{h'} \in heur, h - c \leq h' < \infty$:
    $B' := bdd\text{-}intersection(heur_{h'}, open\text{-}zero)$
    $open_{g+c,h'} := bdd\text{-}union(open_{g+c,h'}, B')$
  return $open_{*,*}$

Heuristics

How can we generate symbolic heuristics?

- Symbolic Pattern Databases
  - Uniform-cost search can easily be adapted to regression search.
  - Can search backwards in abstract transition systems
  - BDD for closed states with (backwards-) g-value i is heuristic BDD for $h = i$.
- Merge-and-Shrink Abstractions
  - Algebraic Decision Diagrams are like BDDs but sink nodes are labeled with arbitrary numbers.
  - Can map states to numbers.
  - Cascading tables of merge-and-shrink heuristics with linear merge strategy can efficiently be transformed into an ADD.
  - Result can be used in symbolic search instead of BDD set.

Discussion

Importance of Variable Ordering

- For good performance, we need a good variable ordering.
  - Variables that refer to the same state variable before and after operator application (v and v') should be neighbors in the transition relation BDD.
- This is important for the performance of *BDD-rename* in the *image* and *pre-image* computation.

Transition Relations in $\mathcal{T}$

- We only required that all operators are represented by some $(T, c) \in \mathcal{T}$ and that the costs are correct.
- Extreme cases:
  - One element $(\tau_V(o), cost(o))$ for each operator o
  - Only one element for each operator cost, covering all operators of that cost.
- Trade-off:
  - Large number of entries leads to large number of image computations.
  - Size of T can grow exponentially with number of covered operators.
- There exist different aggregation strategies.

Performance

- In symbolic planning, blind search is often better than informed search.
- Practical implementations also perform regression or bidirectional search.
- This is only a minor modification of uniform-cost search.

Summary

Summary

- Symbolic search operates on sets of states instead of individual states as in explicit-state search.
- State sets and transition relations can be represented as BDDs.
- A good variable ordering and an efficient image computation are crucial for performance.

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